GCSE MATHEMATICS

Aiming for Grade 7

REVISION BOOKLET

Exam Dates:

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Pixi MATHS

Name: Worked solutions

Contents

Surds

Things to remember:

- √ means square root;
- To simplify surds, find all its factors;
- To rationalise the denominator, find an equivalent fraction where the denominator is rational.

Questions:

1. Work out

$$\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$$

Give your answer in its simplest form.

	5	$+\sqrt{3}$
5	25	$+5\sqrt{3}$
$-\sqrt{3}$	$-5\sqrt{3}$	-3

$\frac{22}{\sqrt{22}} \times \frac{\sqrt{22}}{\sqrt{22}} = \frac{22\sqrt{22}}{22}$

$\sqrt{22}$

(Total 3 marks)

2. (a) Rationalise the denominator of $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\frac{\sqrt{3}}{3}$

(1)

(b) Expand $(2+\sqrt{3})(1+\sqrt{3})$

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

	2	$+\sqrt{3}$
1	2	$+\sqrt{3}$
$+\sqrt{3}$	$+2\sqrt{3}$	$+3$

$5+3\sqrt{3}$

(2)

(Total 3 marks)

3. (a) Rationalise the denominator of $\frac{1}{\sqrt{7}}$

$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

$\frac{\sqrt{7}}{7}$

.....

(2)

(b) (i) Expand and simplify $(\sqrt{3} + \sqrt{15})^2$
Give your answer in the form $a + b\sqrt{3}$ where *a* and *b* are integers.

	$\sqrt{3}$	$+\sqrt{15}$
$\sqrt{3}$	3	$\sqrt{45}$
$+\sqrt{15}$	$\sqrt{45}$	$+15$

$18 + 2\sqrt{45}$
$= 18 + 2\sqrt{9}\sqrt{5}$

$18 + 6\sqrt{5}$

.....

(ii) All measurements on the triangle are in centimetres.
ABC is a right-angled triangle.
k is a positive integer.

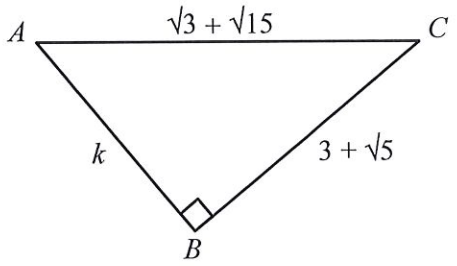


Diagram **NOT** accurately drawn

Find the value of *k*.

$k^2 + (3 + \sqrt{5})^2 = (\sqrt{3} + \sqrt{15})^2$

$k^2 + 9 + 6\sqrt{5} + 5 = 18 + 6\sqrt{5}$

$k^2 = 4$

$k = 2$

k = ...2.....

(5)

(Total 7 marks)

4. Expand and simplify $(\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})$

	$\sqrt{3}$	$-\sqrt{2}$
$\sqrt{3}$	3	$-\sqrt{6}$
$-\sqrt{2}$	$-\sqrt{6}$	$+2$

$5 - 2\sqrt{6}$

(Total 2 marks)

5. (a) Write down the value of $49^{1/2}$ $\sqrt{49}$

7

(1)

(b) Write $\sqrt{45}$ in the form $k\sqrt{5}$, where k is an integer.

$\sqrt{9} \times \sqrt{5}$

$3\sqrt{5}$

(1)
(Total 2 marks)

6. Write $\dfrac{\sqrt{18} + 10}{\sqrt{2}}$ in the form $a + b\sqrt{3}$ where a and b are integers.

$$\frac{\sqrt{2}(\sqrt{18} + 10)}{2} = \frac{\sqrt{36} + 10\sqrt{2}}{2}$$

$$= \frac{6 + 10\sqrt{2}}{2}$$

$$= 3 + 5\sqrt{2}$$

$a =$ 3

$b =$ 5

(Total 2 marks)

7. Expand and simplify $(2 + \sqrt{3})(7 - \sqrt{3})$
Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

	2	$+\sqrt{3}$
7	14	$+7\sqrt{3}$
$-\sqrt{3}$	$-2\sqrt{3}$	-3

$11 + 5\sqrt{3}$

(Total 3 marks)

8. Rationalise the denominator of $\dfrac{(4 + \sqrt{2})(4 - \sqrt{2})}{\sqrt{7}}$
Give your answer in its simplest form.

	4	$+\sqrt{2}$
4	16	$+4\sqrt{2}$
$-\sqrt{2}$	$-4\sqrt{2}$	-2

$\dfrac{14}{\sqrt{7}} = \dfrac{14\sqrt{7}}{7}$

$2\sqrt{7}$

(Total for question = 3 marks)

9. Show that $\dfrac{(4 - \sqrt{3})(4 + \sqrt{3})}{\sqrt{13}}$ simplifies to $\sqrt{13}$

	4	$-\sqrt{3}$
4	16	$-4\sqrt{3}$
$+\sqrt{3}$	$+4\sqrt{3}$	-3

$\dfrac{13}{\sqrt{13}} = \dfrac{13\sqrt{13}}{13} = \sqrt{13}$

(Total for question = 2 marks)

Bounds Calculations

Things to remember:

- Calculating bounds is the opposite of rounding – they are the limits at which you would round up instead of down, and vice versa.
- When dividing bounds, UB = UB ÷ LB and LB = LB ÷ UB

Questions:

1. A piece of wood has a length of 65 centimetres to the nearest centimetre.

(a) What is the least possible length of the piece of wood?

64.5

(1)

(b) What is the greatest possible length of the piece of wood?

65.5

(1)

(Total for Question is 2 marks)

2. Chelsea's height is 168 cm to the nearest cm.

(a) What is Chelsea's minimum possible height?

167.5 cm

(1)

(b) What is Chelsea's maximum possible height?

168.5 cm

(1)

(Total for Question is 2 marks)

3. $I = \frac{V}{R}$

V = 250 correct to the nearest 5

R = 3900 correct to the nearest 100

Work out the lower bound for the value of I.

Give your answer correct to 3 decimal places.

You must show your working.

$LB_I = \frac{LB_V}{UB_R} = \frac{247.5}{3950} = 0.0626\ldots$

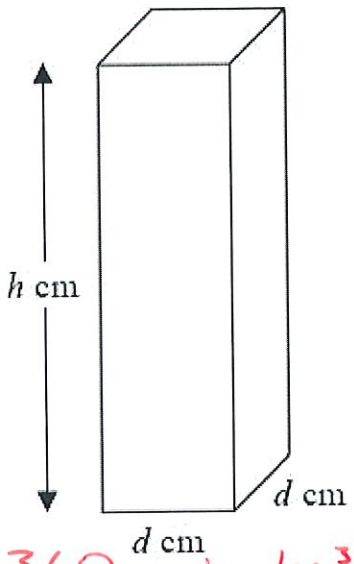
0.063

(Total for question = 3 marks)

4. Here is a solid bar made of metal.
The bar is in the shape of a cuboid.
The height of the bar is h cm.
The base of the bar is a square of side d cm.
The mass of the bar is M kg.

$\frac{M}{D \times V}$

$d = 8.3$ correct to 1 decimal place.
$M = 13.91$ correct to 2 decimal places.
$h = 84$ correct to the nearest whole number.



Find the value of the density of the metal to an appropriate degree of accuracy.
Give your answer in g/cm^3.
You must explain why your answer is to an appropriate degree of accuracy.

$LB_D = \frac{LB_M}{UB_V} = \frac{13.905}{8.35^2 \times 84.5} = 0.002\,360\ldots$ kg/cm^3

$UB_D = \frac{UB_M}{LB_V} = \frac{13.915}{8.25^2 \times 83.5} = 0.002\,448\ldots$ kg/cm^3

0.0024 kg/cm^3 = 2.4 g/cm^3
Both the upper and lower bounds are the same to 2 significant figures.

(Total for question = 5 marks)

5. Steve travelled from Ashton to Barnfield.
He travelled 235 miles, correct to the nearest 5 miles.
The journey took him 200 minutes, correct to the nearest 5 minutes.
Calculate the lower bound for the average speed of the journey.
Give your answer in **miles per hour**, correct to 3 significant figures.
You must show all your working.

$\frac{D}{S \times T}$

$LB_S = \frac{LB_D}{UB_T} = \frac{232.5}{202.5} = 1.148\ldots$ miles per minute

$1.148\ldots \times 60 = 68.88\ldots$ mph

68.9 ..... mph

(Total for question = 4 marks)

6. The value of p is 4.3
The value of q is 0.4
Both p and q are given correct to the nearest 0.1

(a) Write down the lower bound for p.

4.25

(1)

$$r = p + \frac{1}{q}$$

(b) Work out the upper bound for r.
You must show all your working.

$UB_r = UB_p + \frac{1}{LB_q} = 4.35 + \frac{1}{0.35}$

$= 7.2071\ldots$

7.21 (3 s.f.)

(3)

(Total for question = 4 marks)

7. $m = \frac{\sqrt{s}}{t}$ $s = 3.47$ correct to 3 significant figures

$t = 8.132$ correct to 4 significant figures

By considering bounds, work out the value of m to a suitable degree of accuracy.
Give a reason for your answer.

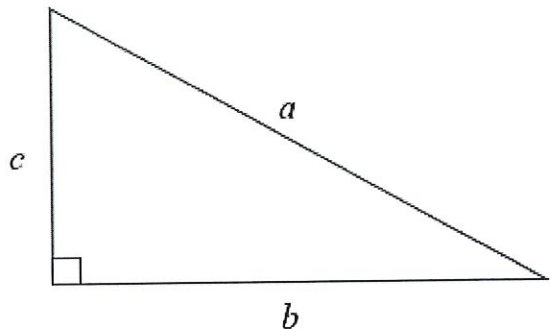
$UB_M = \frac{\sqrt{UB_s}}{LB_t} = \frac{\sqrt{3.475}}{8.1315} = 0.229\,248\ldots$

$LB_M = \frac{\sqrt{LB_s}}{UB_t} = \frac{\sqrt{3.465}}{8.1325} = 0.228\,890\ldots$

$M = 0.229$ as this is correct for both the upper and lower bound.

(Total for question = 5 marks)

8. *a* is 8.3 cm correct to the nearest mm
b is 6.1 cm correct to the nearest mm



Calculate the upper bound for *c*.
You must show your working.

$c = \sqrt{a^2 - b^2}$

$UB_c = \sqrt{UB_a^2 - LB_b^2}$

$= \sqrt{8.35^2 - 6.05^2}$

$= 5.754\,997\ldots$

5.75 (3 s.f.) ..... cm

(Total for question = 4 marks)

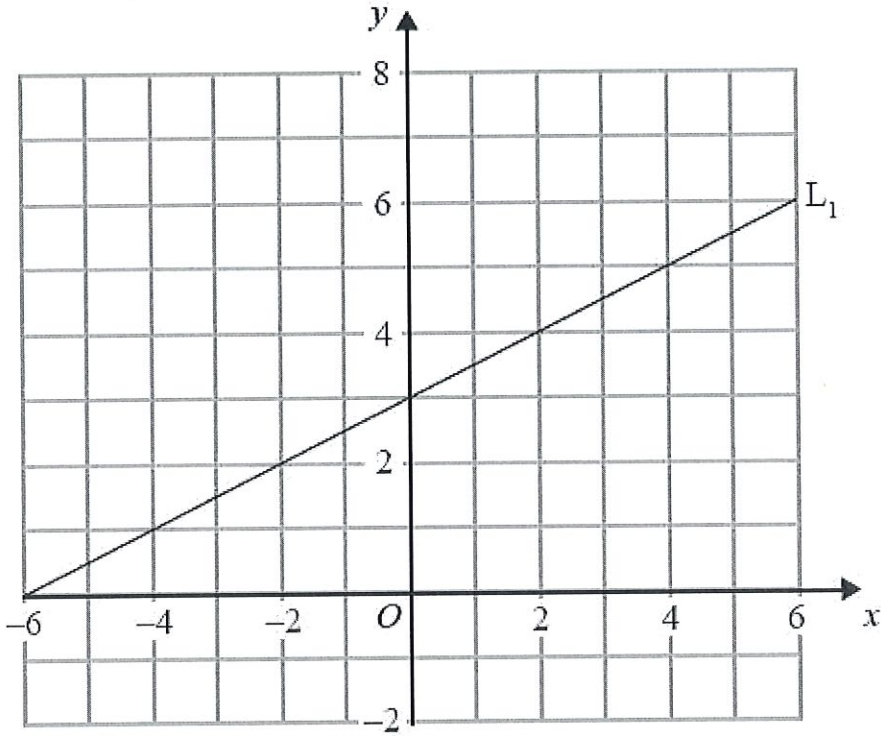
Parallel and Perpendicular Graphs

Things to remember:

- The general equation of a linear graph is given by y = mx + c, where m is the gradient and c is the y-intercept.
- Parallel graphs have the same gradient.
- Gradients of perpendicular graphs have a product of -1.

Questions:

1. The diagram shows a straight line, L_1, drawn on a grid.



A straight line, L_2, is parallel to the straight line L_1 and passes through the point (0, −5).
Find an equation of the straight line L_2.

$y = mx + c$

$m = \frac{1}{2}$

$y = \frac{1}{2}x + c$

$y = \frac{1}{2}x - 5$

(Total for Question is 3 marks)

2. The straight line **L** has equation $y = 2x - 5$
Find an equation of the straight line perpendicular to **L** which passes through (−2, 3).

$m_1 = 2 \Rightarrow m_2 = -\frac{1}{2}$

$y = -\frac{1}{2}x + c \quad (-2, 3)$

$3 = -\frac{1}{2}(-2) + c$

$2 = c$

$y = -\frac{1}{2}x + 2$

(Total for Question is 3 marks)

3. In the diagram, *ABC* is the line with equation $y = -\frac{1}{2}x + 5$
AB = *BC*
D is the point with coordinates (−13, 0)

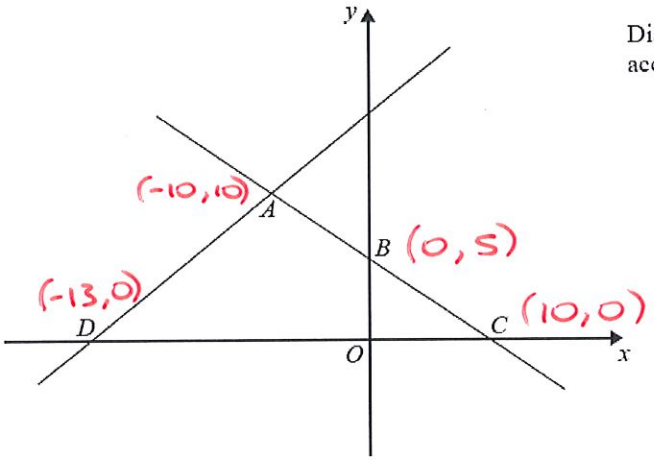


Diagram **NOT** accurately drawn

Find an equation of the line through *A* and *D*.

$y = mx + c$

$m = \frac{10 - 0}{-10 + 13} = \frac{10}{3}$

$y = \frac{10}{3}x + c \quad (-10, 10)$

$10 = \frac{10}{3}(-10) + c$

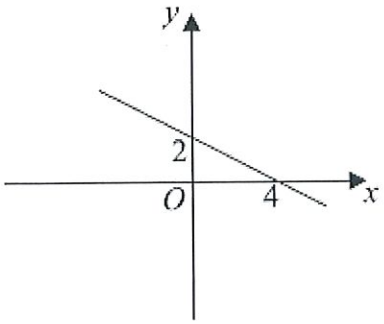
$\frac{50}{3} = c$

$y = \frac{10}{3}x + \frac{50}{3}$

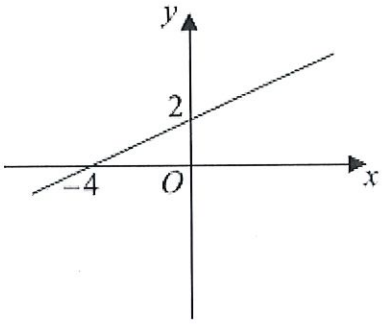
(Total for question = 5 marks)

4. Here are the graphs of 6 straight lines.

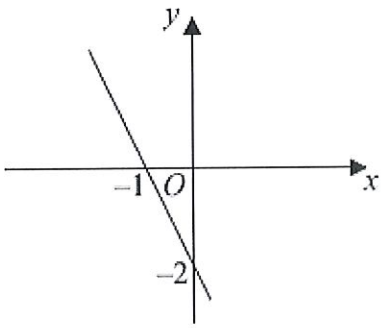
Graph **A**



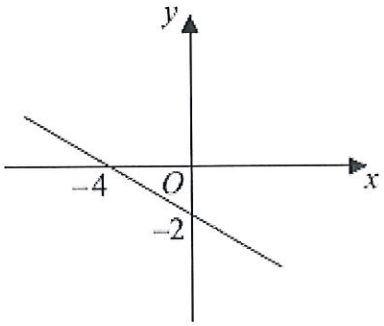
Graph **B**



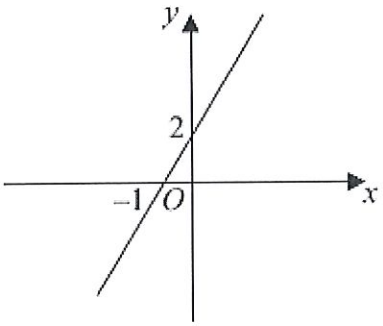
Graph **C**



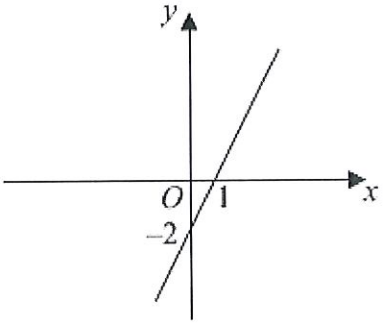
Graph **D**



Graph **E**



Graph **F**



Match each of the graphs **A**, **B**, **C**, **D**, **E** and **F** to the equations in the table.

Equation	$y = \frac{1}{2}x + 2$	$y = 2x - 2$	$y = -\frac{1}{2}x + 2$	$y = -2x - 2$	$y = 2x + 2$	$y = -\frac{1}{2}x - 2$
Graph	B	F	A	C	E	D

(Total for Question is 3 marks)

5. In the diagram, A is the point (-2, 0)
B is the point (0, 4)
C is the point (5, -1)

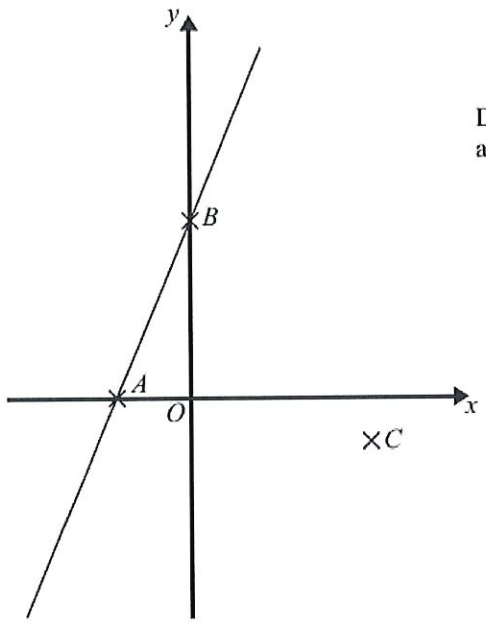


Diagram **NOT** accurately drawn

Find an equation of the line that passes through C and is perpendicular to AB.

$m_1 = \frac{4-0}{0+2} = \frac{4}{2} = 2 \Rightarrow m_2 = -\frac{1}{2}$

$y = mx + c$

$y = -\frac{1}{2}x + c \quad (5, -1)$

$-1 = -\frac{1}{2}(5) + c$

$\frac{3}{2} = c$

$y = -\frac{1}{2}x + \frac{3}{2}$

(Total for Question is 4 marks)

6. Find an equation of the straight line that is perpendicular to the straight line $x + 2y = 5$ and that passes through the point (3, 7).

$x + 2y = 5$

$2y = -x + 5$

$y = -\frac{1}{2}x + \frac{5}{2}$

$m_1 = -\frac{1}{2} \Rightarrow m_2 = 2$

$y = mx + c$

$y = 2x + c \quad (3, 7)$

$7 = 2(3) + c$

$1 = c$

$y = 2x + 1$

(Total for Question is 4 marks)

*7. **A** and **B** are straight lines.
Line **A** has equation $2y = 3x + 8$
Line **B** goes through the points $(-1, 2)$ and $(2, 8)$

Do lines **A** and **B** intersect?
You must show all your working.

$2y = 3x + 8$

$y = \frac{3}{2}x + 4 \quad \Rightarrow m_A = \frac{3}{2}$

$m_B = \frac{8-2}{2+1} = \frac{6}{3} = 2$

Since the gradients are not equal, the lines are not parallel and therefore will intersect.

(Total for Question is 3 marks)

8. A straight line, ***L***, is perpendicular to the line with equation $y = 1 - 3x$.
The point with coordinates $(6, 3)$ is on the line ***L***.
Find an equation of the line ***L***.

$m_1 = -3$

$m_2 = \frac{1}{3}$

$y = mx + c$

$y = \frac{1}{3}x + c \quad (6, 3)$

$3 = \frac{1}{3}(6) + c$

$c = 1$

$y = \frac{1}{3}x + 1$

(Total for Question is 3 marks)

Transformations of graphs

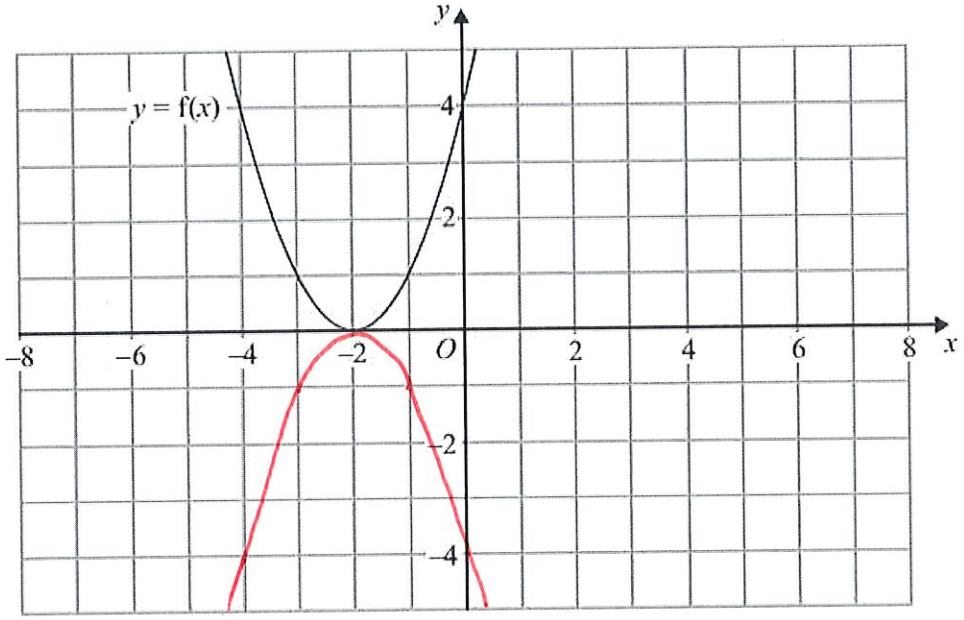
Things to remember:

- f(x) means the function of x.
- -f(x) is a reflection in the x-axis.
- f(-x) is a reflection in the y-axis.
- f(x – a) is a translation in the x-axis, a units.
- f(x) + b is a translation in the y-axis, b units.
- cf(x) is an enlargement in the y-axis, scale factor c.
- f(dx) is an enlargement in the x-axis, scale factor $\frac{1}{d}$.

Questions:

1. $y = f(x)$

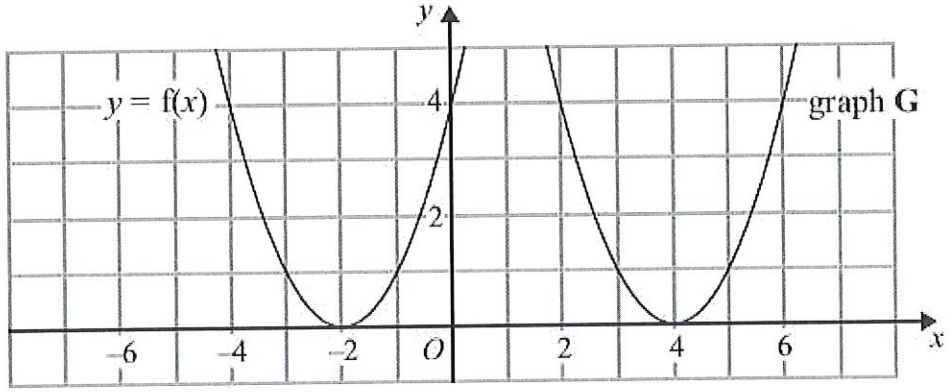
The graph of $y = f(x)$ is shown on the grid.



(a) On the grid above, sketch the graph of $y = -f(x)$.

(2)

The graph of $y = f(x)$ is shown on the grid.



The graph **G** is a translation of the graph of $y = f(x)$.

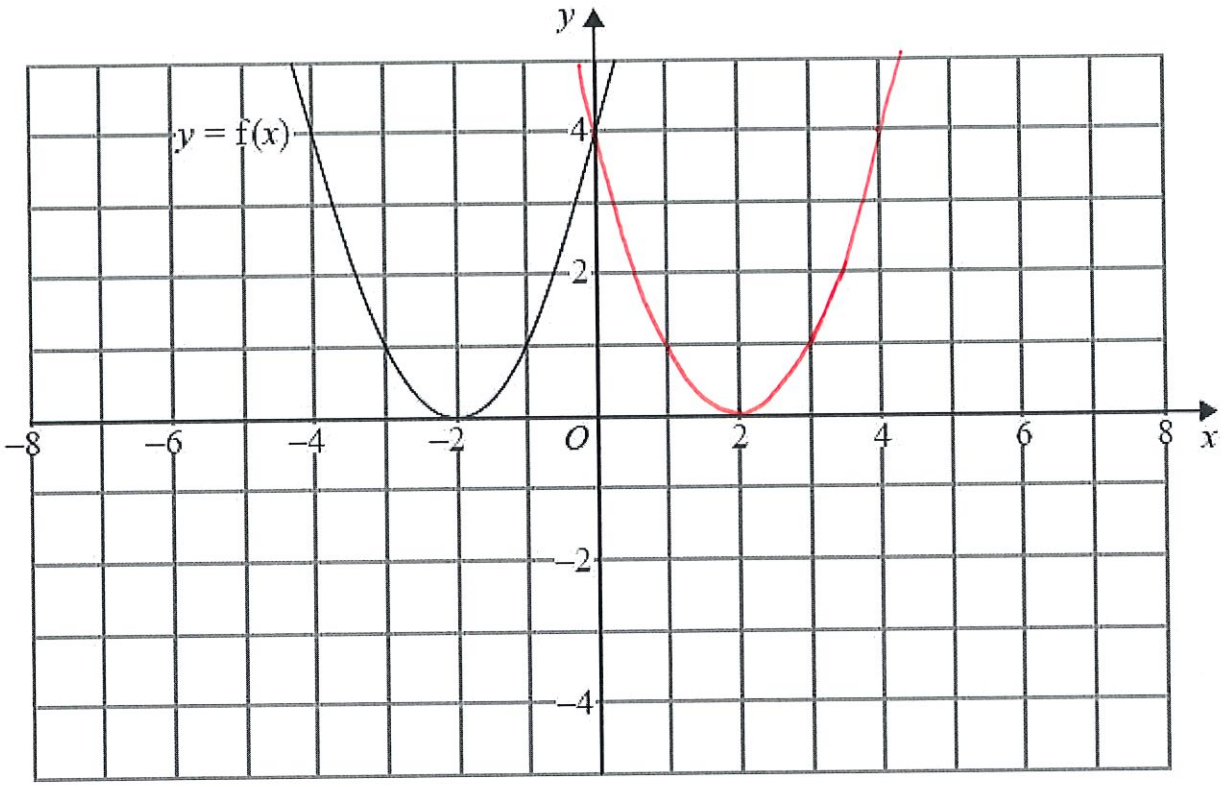
(b) Write down the equation of graph **G**.

$y = f(x - 6)$

(2)

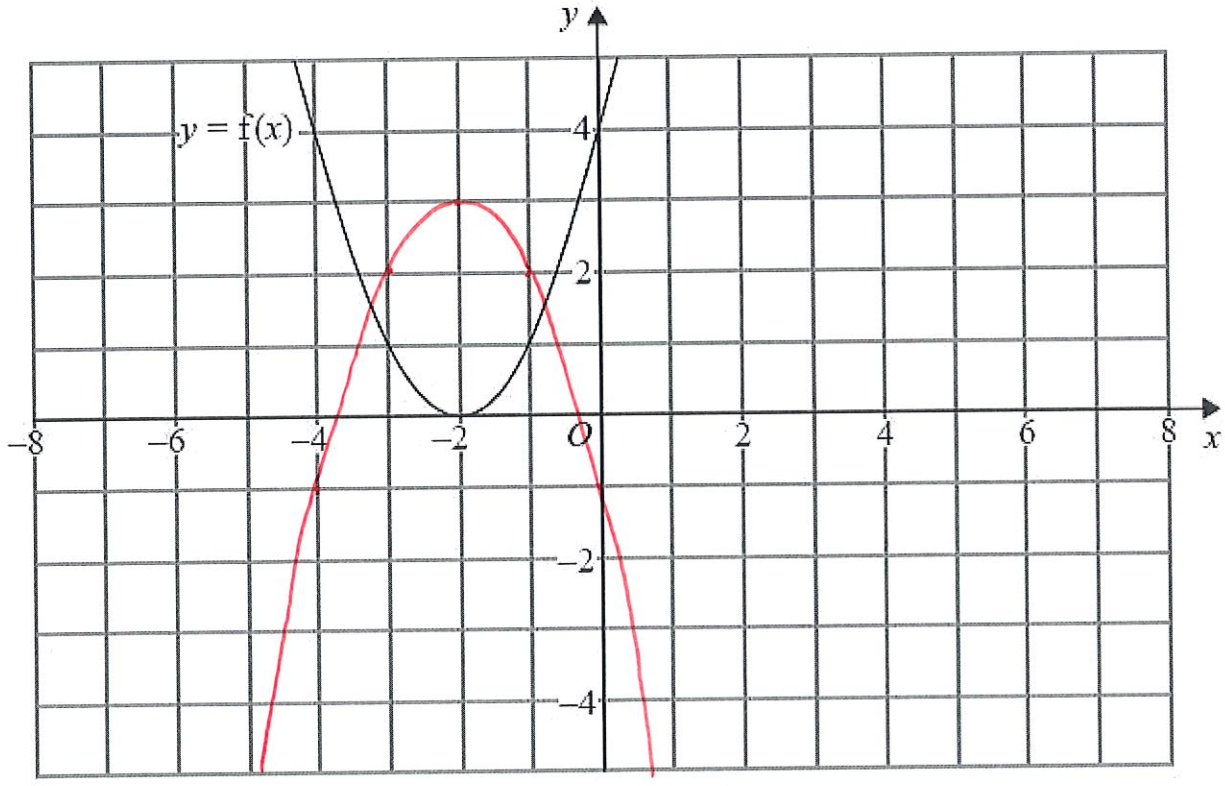
(Total for Question is 3 marks)

2. The graph of $y = f(x)$ is shown on both grids below.



(a) On the grid above, sketch the graph of $y = f(-x)$

(1)



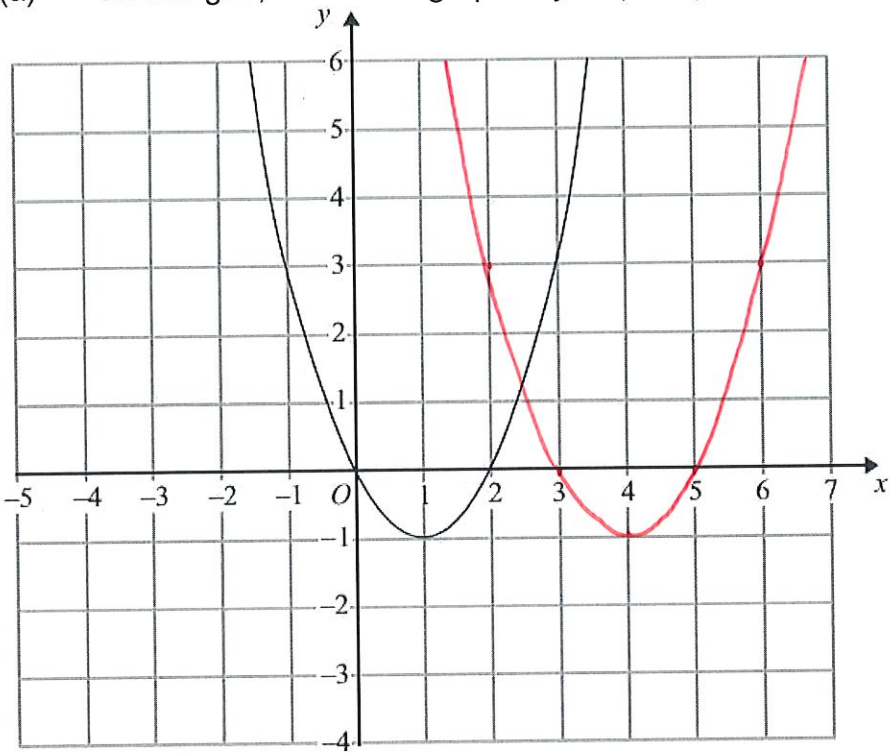
(b) On this grid, sketch the graph of $y = -f(x) + 3$

(1)

(Total for question = 2 marks)

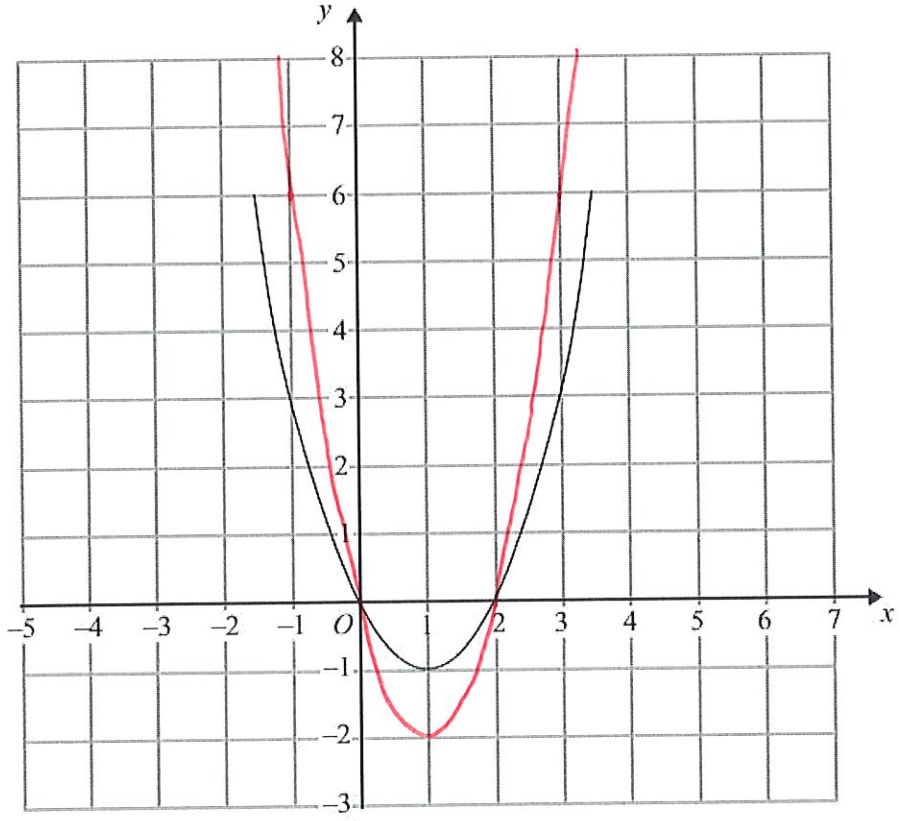
3. The graph of $y = f(x)$ is shown on each of the grids.

(a) On this grid, sketch the graph of $y = f(x - 3)$



(2)

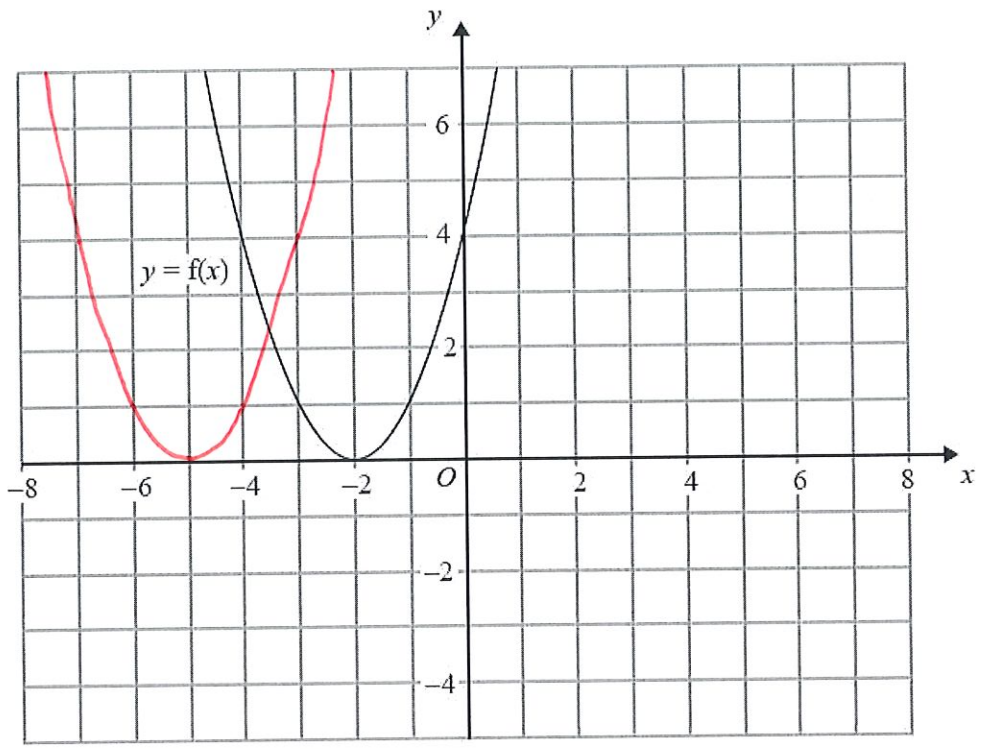
(b) On this grid, sketch the graph of $y = 2f(x)$



(2)

(Total for Question is 4 marks)

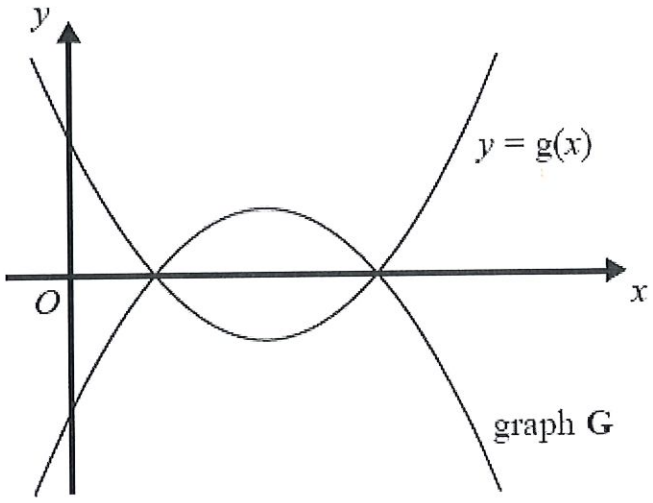
4. The graph of $y = f(x)$ is shown on the grid.



(a) On the grid above, sketch the graph of $y = f(x + 3)$

(2)

The graph of $y = g(x)$ is shown below.



The graph **G** is the reflection of $y = g(x)$ in the x-axis.

(b) Write down an equation of graph **G**.

$y = -g(x)$

(1)

(Total for question = 3 marks)

Algebraic Fractions – Simplifying

Things to remember:

- Factorise the numerator and denominator;
- Cancel common factors;
- Then add/subtract/multiply divide if necessary.

Questions:

1. Simplify $\frac{p^2-9}{2p+6}$ $= \frac{(p+3)(p-3)}{2(p+3)}$

$\frac{p-3}{2}$

.....

(Total 3 marks)

2. Simplify fully $\frac{6x^2+3x}{4x^2-1}$ $= \frac{3x(2x+1)}{(2x+1)(2x-1)}$

$\frac{3x}{2x-1}$

.....

(Total 3 marks)

3. Simplify $\frac{x^2+2x+1}{x^2+3x+2}$ $= \frac{(x+1)(x+1)}{(x+1)(x+2)}$

$\frac{x+1}{x+2}$

.....

(Total 3 marks)

4. Simplify fully $\dfrac{x^2+x-6}{x^2-7x+10}$ $= \dfrac{(x+3)(x-2)}{(x-5)(x-2)}$

$\dfrac{x+3}{x-5}$

.....

(Total 3 marks)

5. Simplify fully $\dfrac{x^2-8x+15}{2x^2-7x-15}$ $= \dfrac{(x-3)(x-5)}{(2x+3)(x-5)}$

-30

	x	-5
$2x$	$2x^2$	$-10x$
$+3$	$+3x$	-15

$\dfrac{x-3}{2x+3}$

.....

(Total 3 marks)

6. Simplify fully $\dfrac{2x^2+3x+1}{x^2-3x-4}$ $= \dfrac{(2x+1)(x+1)}{(x-4)(x+1)}$

	x	1
$2x$	$2x^2$	$2x$
1	x	$+1$

$\dfrac{2x+1}{x-4}$

.....

(Total 3 marks)

7. (a) Simplify $\dfrac{2x+4}{x^2+4x+4}$ $= \dfrac{2(x+2)}{(x+2)(x+2)}$

$\dfrac{2}{x+2}$

.....

(3)

(b) Write $\dfrac{1}{x+4} + \dfrac{2}{x-4}$ as a single fraction in its simplest form.

$\dfrac{x-4+2(x+4)}{(x+4)(x-4)} = \dfrac{x-4+2x+8}{x^2-16}$

$\dfrac{3x+4}{x^2-16}$

.....

(3)

(Total 6 marks)

8. Simplify fully $\dfrac{x+3}{4} + \dfrac{x-5}{3}$

$\dfrac{3(x+3)+4(x-5)}{12} = \dfrac{3x+9+4x-20}{12}$

$\dfrac{7x-11}{12}$

.....

(Total 3 marks)

Algebraic fractions – solving

Things to remember:

- Multiply every term by the product of the denominators;
- Solve to find x.

Questions:

1. Solve $\frac{5(2x+1)}{3} = 4x + 7$

$5(2x+1) = 3(4x+7)$

$10x + 5 = 12x + 21$

$-16 = 2x$

$-8 = x$

$x =$ ..... -8 .....

(Total 3 marks)

2. (a) Show that the equation $\frac{5}{x+2} = \frac{4-3x}{x-1}$

can be rearranged to give $3x^2 + 7x - 13 = 0$

$5(x-1) = (4-3x)(x+2)$

$5x - 5 = 4x - 3x^2 + 8 - 6x$

$3x^2 + 7x - 13 = 0$

(3)

(b) Solve $3x^2 + 7x - 13 = 0$

Give your solutions correct to 2 decimal places.

$x =$ ..... 1.22 ..... or $x =$ ..... -3.55 .....

(3)

(Total 6 marks)

3. Solve the equation $\frac{x}{2x-3} + \frac{4}{x+1} = 1$

$x(x+1) + 4(2x-3) = (x+1)(2x-3)$

$x^2 + x + 8x - 12 = 2x^2 + 2x - 3x - 3$

$0 = x^2 - 10x + 9$

$0 = (x-1)(x-9)$

$x =$ 1 or 9

(Total 5 marks)

4. Solve the equation $\frac{3}{x+3} - \frac{4}{x-3} = \frac{5x}{x^2-9}$

$3(x-3) - 4(x+3) = \frac{5x(x+3)(x-3)}{(x+3)(x-3)}$

$3x - 9 - 4x - 12 = 5x$

$-21 = 6x$

$-3.5 = x$

$x =$ −3.5

(Total 4 marks)

5. (a) Solve $\frac{3}{x} + \frac{3}{2x} = 2$

$3(2x) + 3x = 2(x)(2x)$

$6x + 3x = 4x^2$

$0 = 4x^2 - 9x$

$0 = x(4x - 9)$

x= 0 or 9/4

(2)

(b) Using your answer to part (a), or otherwise, solve $\frac{3}{(y-1)^2} + \frac{3}{2(y-1)^2} = 2$

$(y-1)^2 = 0$ or $(y-1)^2 = 9/4$

$y - 1 = 0$ $y - 1 = \pm 3/2$

$y = 1$ $y = 5/2$ or $-1/2$

y = 1 or y = 5/2 or y = -1/2

(3)

(Total 5 marks)

Solving Quadratic Inequalities

Things to remember:

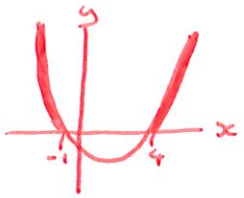
- Start by solving the quadratic to find the values of x, then sketch the graph to determine the inequality.

Questions:

1. Solve $x^2 > 3x + 4$

$x^2 - 3x - 4 > 0$

$(x - 4)(x + 1) > 0$



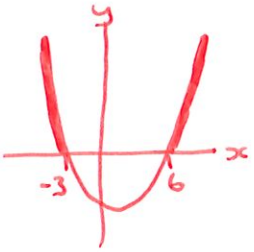
$x < -1$ or $x > 4$

(Total for question = 3 marks)

2. Solve the inequality $x^2 > 3(x + 6)$

$x^2 - 3x - 18 > 0$

$(x - 6)(x + 3) > 0$



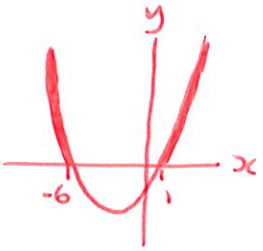
$x < -3$ or $x > 6$

(Total for question = 4 marks)

3. Solve the inequality $x^2 + 5x > 6$

$x^2 + 5x - 6 > 0$

$(x + 6)(x - 1) > 0$

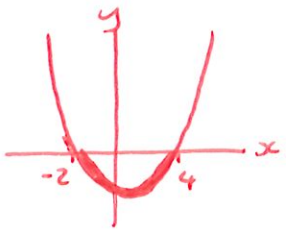


$x < -6$ or $x > 1$

(Total for question = 3 marks)

4. Solve the inequality $x^2 - 2x - 8 < 0$

$(x + 2)(x - 4) < 0$



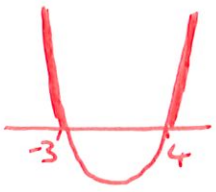
$-2 < x < 4$

(Total for question = 3 marks)

5. Solve the inequality $x^2 - x \geq 12$

$x^2 - x - 12 \geqslant 0$

$(x - 4)(x + 3) \geqslant 0$



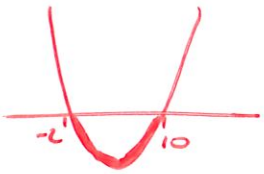
$x < -3$ or $x > 4$

(Total for question = 3 marks)

6. Solve the inequality $x^2 \leq 4(2x + 5)$

$x^2 - 8x - 20 \leqslant 0$

$(x - 10)(x + 2) \leqslant 0$

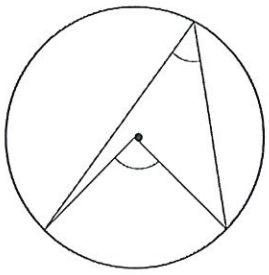


$-2 \leqslant x \leqslant 10$

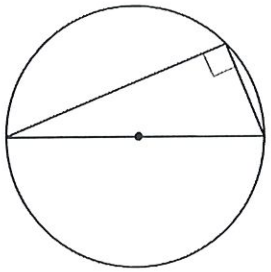
(Total for question = 4 marks)

Circle theorems

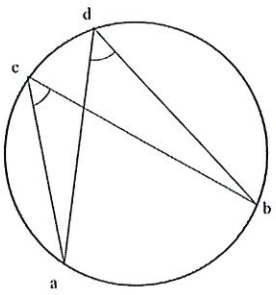
Things to remember:



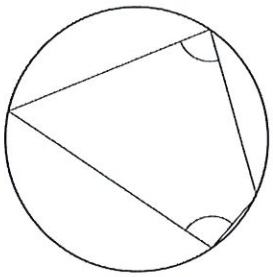
The angle at the centre is twice the angle at the circumference.



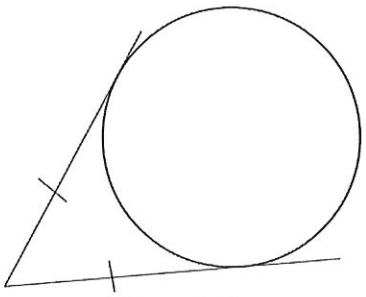
The angle in a semi-circle is 90°.



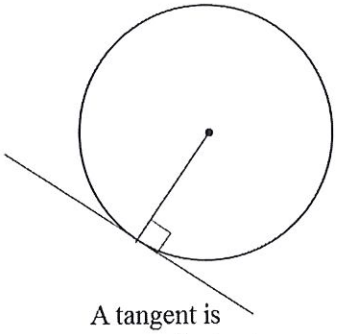
Angles subtended by the same arc are equal.



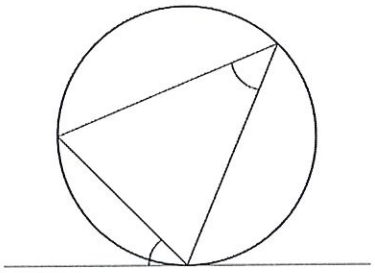
Opposite angles in a cyclic quadrilateral sum to 180°.



Tangents from a point are equal.



A tangent is perpendicular to a radius.



Angles in alternate segments are equal.

Questions:

1.

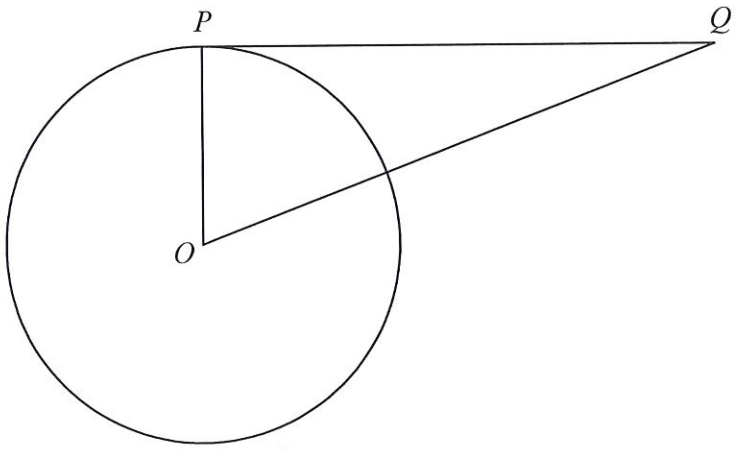


Diagram **NOT** accurately drawn

P is a point on the circumference of the circle, centre *O*.

PQ is a tangent to the circle.

(i) Write down the size of angle *OPQ*.

90 °

(ii) Give a reason for your answer.

A tangent is perpendicular to the radius.

(Total 2 marks)

2.

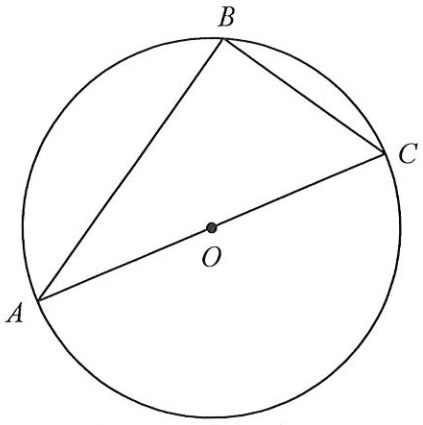


Diagram **NOT** accurately drawn

A, *B* and *C* are points on the circumference of a circle, centre *O*.

AC is a diameter of the circle.

(a) (i) Write down the size of angle *ABC*.

90 °

(ii) Give a reason for your answer.

Angles in semi-circles are 90°.

(2)

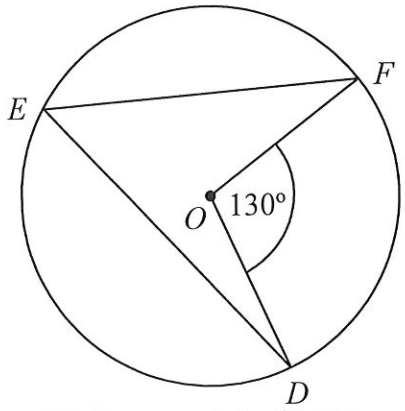


Diagram **NOT** accurately drawn

D, *E* and *F* are points on the circumference of a circle, centre *O*.

Angle *DOF* = 130°.

(b) (i) Work out the size of angle *DEF*.

65 °

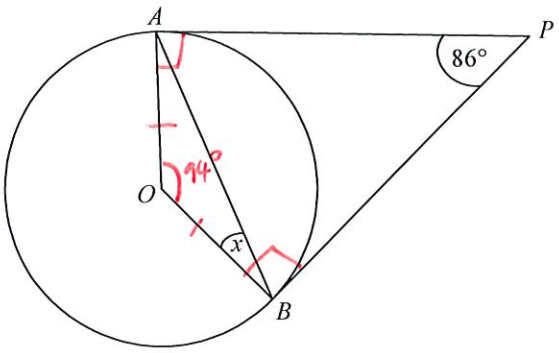
(ii) Give a reason for your answer.

Angles subtended by the same arc are halved at the circumference.

(2)

(Total 4 marks)

3.



180 − 94 / 2

Diagram **NOT** accurately drawn
A and *B* are points on the circumference of a circle, centre *O*.
PA and *PB* are tangents to the circle.
Angle *APB* is 86°.
Work out the size of the angle marked *x*.

43 .....°

(Total 2 marks)

4.

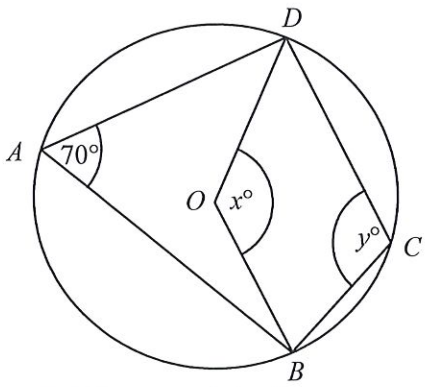


Diagram **NOT** accurately drawn
In the diagram, *A*, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
Angle *BAD* = 70°.
Angle *BOD* = *x*°.
Angle *BCD* = *y*°.

(a) (i) Work out the value of *x*.

140 .....°

(ii) Give a reason for your answer.

Angles subtended by same arc are doubled at the centre.

(2)

(b) (i) Work out the value of *y*.

110 .....°

(ii) Give a reason for your answer.

Opposite angles in a cyclic quadrilateral sum to 180°.

(2)

(Total 4 marks)

5.

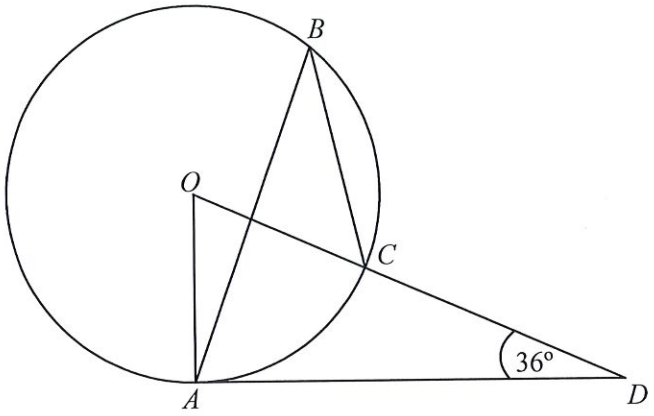


Diagram **NOT** accurately drawn
The diagram shows a circle centre *O*.
A, *B* and *C* are points on the circumference.
DCO is a straight line.
DA is a tangent to the circle.
Angle *ADO* = 36°

(a) Work out the size of angle *AOD*.

54 °

(2)

(b) (i) Work out the size of angle *ABC*.

27 °

(ii) Give a reason for your answer.

Angles subtended by same arc are halved at the circumference.

(3)

(Total 5 marks)

Vectors

Things to remember:

- Use the letter provided in the question.
- Going against the arrow is a negative.
- Vectors need to be written in bold or underlined.
- They can be manipulated similarly to algebra.

Questions:

1. The diagram shows a regular hexagon *ABCDEF* with centre *O*.

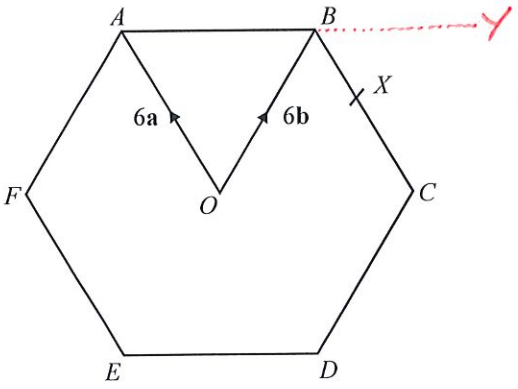


Diagram **NOT** accurately drawn

$\overrightarrow{OA}$ = 6**a** $\overrightarrow{OB}$ = 6**b**

(a) Express in terms of **a** and/or **b**

(i) $\overrightarrow{AB}$,

$6\underline{b} - 6\underline{a}$

(ii) $\overrightarrow{EF}$.

$6\underline{a}$

(2)

X is the midpoint of *BC*.

(b) Express $\overrightarrow{EX}$ in terms of **a** and/or **b**

$12\underline{b} + \frac{1}{2}(-6\underline{a})$

$12\underline{b} - 3\underline{a}$

(2)

Y is the point on *AB* extended, such that *AB* : *BY* = 3:2

(c) Prove that *E*, *X* and *Y* lie on the same straight line.

$\overrightarrow{EY} = 12\underline{b} + \frac{2}{3}(6\underline{b} - 6\underline{a})$

$= 16\underline{b} - 4\underline{a}$

$= 4(4\underline{b} - \underline{a})$

$\overrightarrow{EX} = 12\underline{b} - 3\underline{a}$

$= 3(4\underline{b} - \underline{a})$

Since $(4\underline{b} - \underline{a})$ is a factor of both vectors, they are parallel. Since both vectors share point E, they are on the same line.

(3)

(Total 7 marks)

2. T is the point on PQ for which $PT : TQ = 2 : 1$.

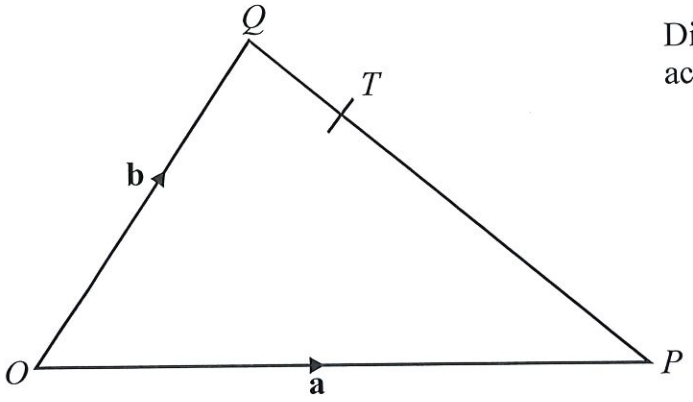


Diagram **NOT** accurately drawn

OPQ is a triangle.

$\overrightarrow{OP}$ = **a** and $\overrightarrow{OQ}$ = **b**.

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{PQ}$.

$\overrightarrow{PQ}$ = $\underline{b} - \underline{a}$

(1)

(b) Express $\overrightarrow{OT}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{OT} = \overrightarrow{OQ} + \frac{1}{3}\overrightarrow{QP}$

$= \underline{b} + \frac{1}{3}(\underline{a} - \underline{b})$

$\frac{1}{3}(2\underline{b} + \underline{a})$

or

$\overrightarrow{OT}$ = $\frac{2}{3}\underline{b} + \frac{1}{3}\underline{a}$

(2)

(Total 3 marks)

3. *OABC* is a parallelogram.

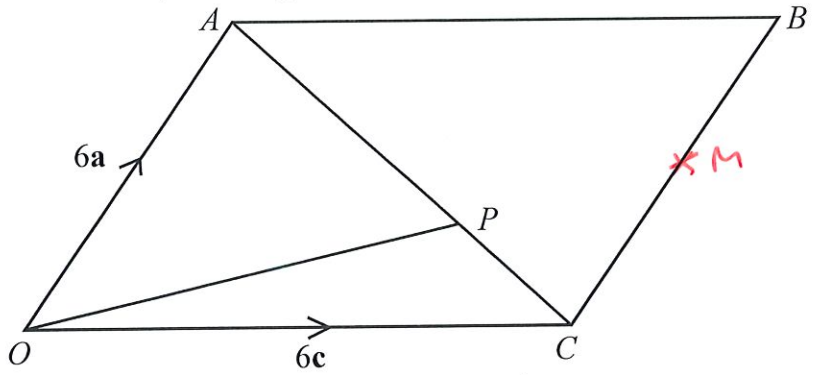


Diagram **NOT** accurately drawn

P is the point on AC such that $AP = \frac{2}{3} AC$.

$\overrightarrow{OA} = 6\mathbf{a}$. $\overrightarrow{OC} = 6\mathbf{c}$.

(a) Find the vector $\overrightarrow{OP}$.
Give your answer in terms of **a** and **c**.

$\overrightarrow{OP} = \overrightarrow{OC} + \frac{1}{3}(\overrightarrow{CA})$

$= 6\underline{c} + \frac{1}{3}(6\underline{a} - 6\underline{c})$

$= 4\underline{c} + 2\underline{a}$

$2\underline{a} + 4\underline{c}$

(3)

The midpoint of CB is M.

(b) Prove that OPM is a straight line.

$\overrightarrow{OP} = 2\underline{a} + 4\underline{c}$

$= 2(\underline{a} + 2\underline{c})$

$\overrightarrow{OM} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{CB}$

$= 6\underline{c} + 3\underline{a}$

$= 3(\underline{a} + 2\underline{c})$

Since $(\underline{a} + 2\underline{c})$ is a common factor, OP and OM are parallel. Point O is shared ∴ same straight line.

(2)

(Total 5 marks)

4. *OPQ* is a triangle.
R is the midpoint of *OP*.
S is the midpoint of *PQ*.
$\overrightarrow{OP}$ = **p** and $\overrightarrow{OQ}$ = **q**

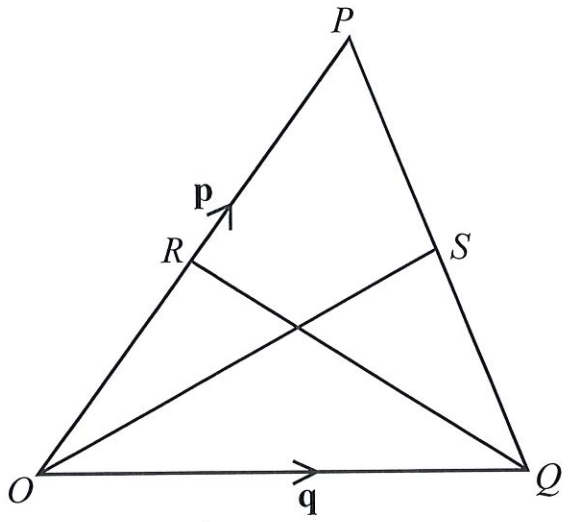


Diagram **NOT** accurately drawn

(i) Find $\overrightarrow{OS}$ in terms of **p** and **q**.

$\vec{OS} = \vec{OQ} + \frac{1}{2}\vec{QP}$
$= q + \frac{1}{2}(p - q)$

$\overrightarrow{OS}$ = $\frac{1}{2}(p + q)$

(ii) Show that *RS* is parallel to *OQ*.

$\vec{OQ} = q$
$\vec{RS} = \frac{1}{2}\vec{OP} + \frac{1}{2}(\vec{PQ})$
$= \frac{1}{2}p + \frac{1}{2}(q - p)$
$= \frac{1}{2}q$

Since q is a factor of both vectors, they are parallel.

(Total 5 marks)

5. $OPQR$ is a trapezium with PQ parallel to OR.

$\overrightarrow{OP} = 2\mathbf{b}$ $\overrightarrow{PQ} = 2\mathbf{a}$ $\overrightarrow{OR} = 6\mathbf{a}$

M is the midpoint of PQ and N is the midpoint of OR.

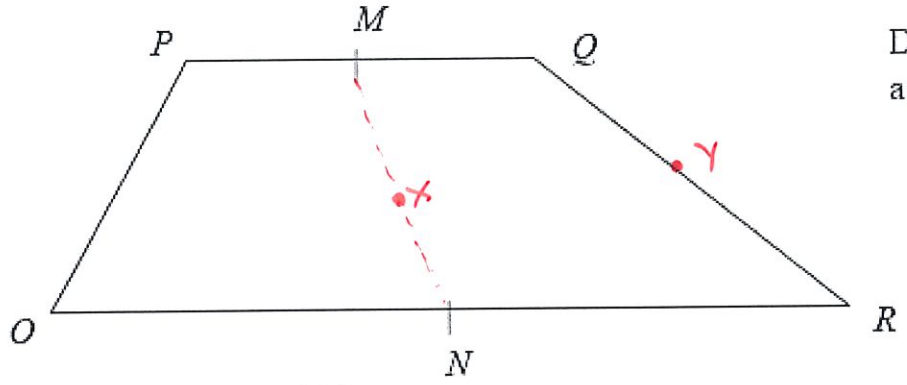


Diagram **NOT** accurately drawn

(a) Find the vector $\overrightarrow{MN}$ in terms of **a** and **b**.

$\vec{MN} = \frac{1}{2}\vec{QP} + \vec{PO} + \frac{1}{2}\vec{ON}$

$= -\underline{a} - 2\underline{b} + 3\underline{a}$

$\overrightarrow{MN} =$ $2\underline{a} - 2\underline{b}$

(2)

X is the midpoint of MN and Y is the midpoint of QR.

(b) Prove that XY is parallel to OR.

$\vec{OR} = 6\underline{a}$

$\vec{XY} = \frac{1}{2}\vec{MN} + \frac{1}{2}\vec{OR} + \frac{1}{2}\vec{RQ}$

$= \underline{a} - \underline{b} + 3\underline{a} + \frac{1}{2}(2\underline{b} - 4\underline{a})$

$= 2\underline{a}$

Since $\underline{a}$ is a factor of both vectors, they are parallel.

(2)

(Total 4 marks)

6. *ABCD* is a straight line.

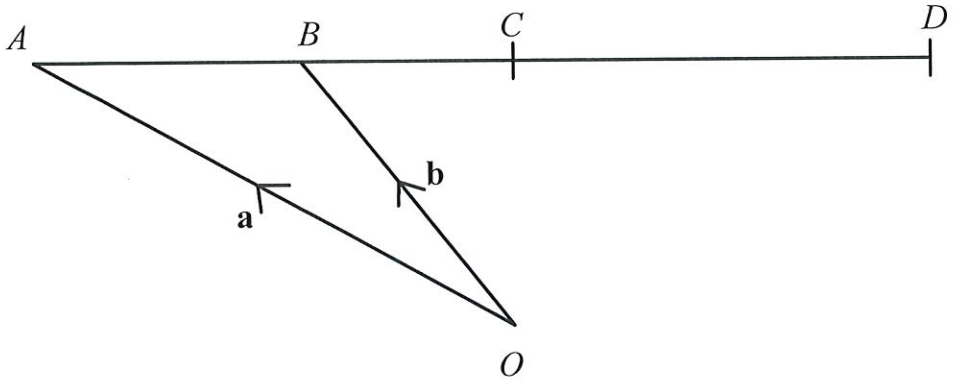


Diagram **NOT** accurately drawn

O is a point so that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

B is the midpoint of *AC*.

C is the midpoint of *AD*.

Express, in terms of **a** and **b**, the vectors

(i) $\overrightarrow{AC}$

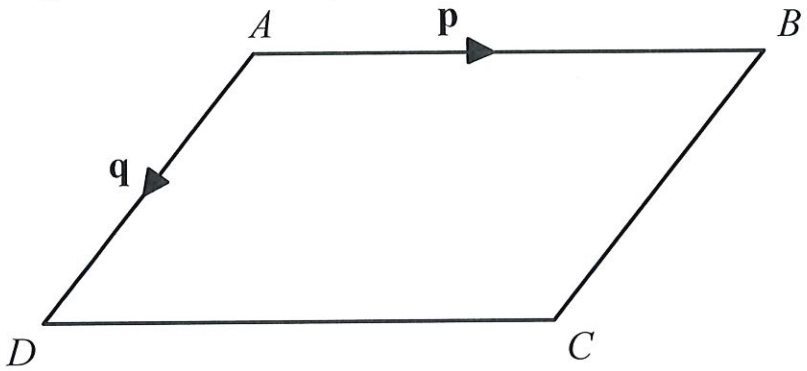
2b - 2a

(ii) $\overrightarrow{OD}$

4b - 4a

(Total 3 marks)

7. Diagram **NOT** accurately drawn



ABCD is a parallelogram.
AB is parallel to *DC*.
AD is parallel to *BC*.

$\overrightarrow{AB}$ = **p**

$\overrightarrow{AD}$ = **q**

(a) Express, in terms of p and q

(i) $\overrightarrow{AC}$

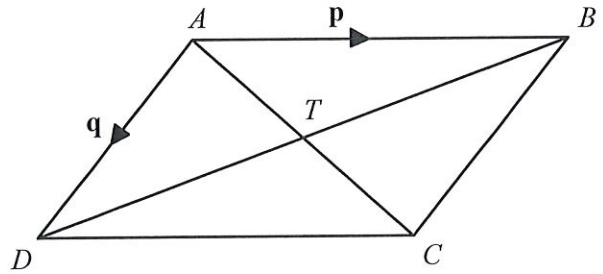
p + q

(ii) $\overrightarrow{BD}$

q − p

(2)

Diagram **NOT** accurately drawn



AC and *BD* are diagonals of parallelogram *ABCD*.
AC and *BD* intersect at *T*.

(b) Express $\overrightarrow{AT}$ in terms of **p** and **q.**

$\overrightarrow{AT} = \frac{1}{2}\overrightarrow{AC}$

$\frac{1}{2}(p+q)$

(1)
(Total 3 marks)

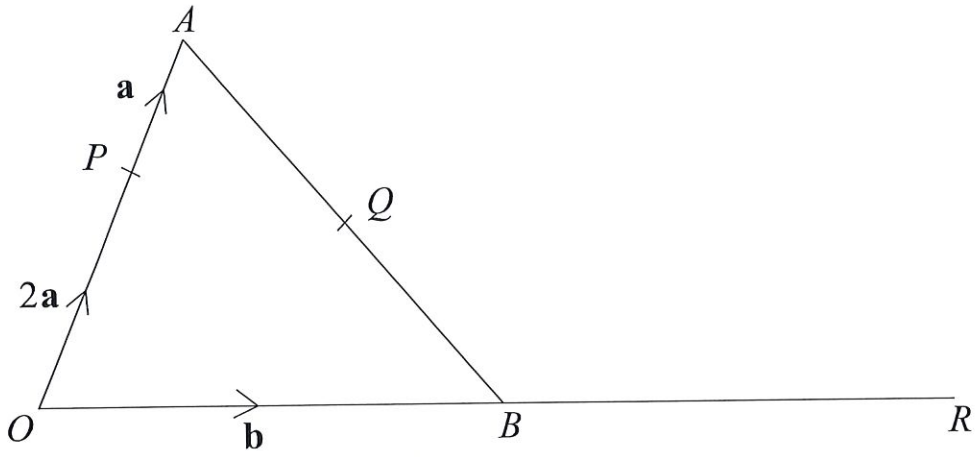
8. Diagram **NOT** accurately drawn

OAB is a triangle.

B is the midpoint of OR.

Q is the midpoint of AB.

$\overrightarrow{OP} = 2\mathbf{a}$ $\overrightarrow{PA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$



(a) Find, in terms of **a** and **b**, the vectors

(i) $\overrightarrow{AB}$,

$\underline{b} - 3\underline{a}$

(ii) $\overrightarrow{PR}$,

$2\underline{b} - 2\underline{a}$

(iii) $\overrightarrow{PQ}$.

$\underline{a} + \frac{1}{2}(\underline{b} - 3\underline{a})$

$\underline{a} + \frac{1}{2}\underline{b} - \frac{3}{2}\underline{a}$

$\frac{1}{2}(\underline{b} - \underline{a})$

(4)

(b) Hence explain why PQR is a straight line.

$(\underline{b} - \underline{a})$ is a factor of both, therefore PR and PQ are parallel. Both vectors share point P so are on a straight line.

(2)

The length of PQ is 3 cm.

(c) Find the length of PR.

$4 \times 3 = 12$

12 cm

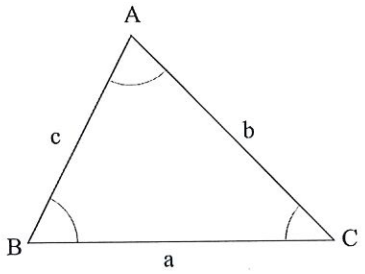
(1)

(Total 7 marks)

Sine and Cosine Rules

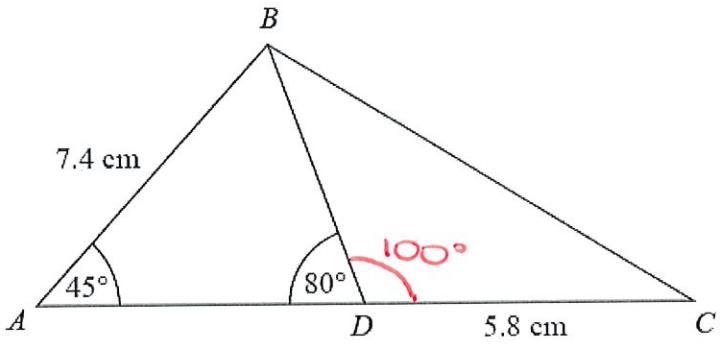
Things to remember:

- For any triangle ABC, $a^2 = b^2 + c^2 - 2bc\cos A$
- For any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- For any triangle ABC, area = ½ a b sinC



Questions:

1. Diagram **NOT** accurately drawn
ABC is a triangle.
D is a point on *AC*.
Angle *BAD* = 45°
Angle *ADB* = 80°
AB = 7.4 cm
DC = 5.8 cm
Work out the length of *BC*.
Give your answer correct to 3 significant figures.



$\frac{BD}{\sin 45} = \frac{7.4}{\sin 80}$ $\therefore$ BD = 5.31... cm (→A)

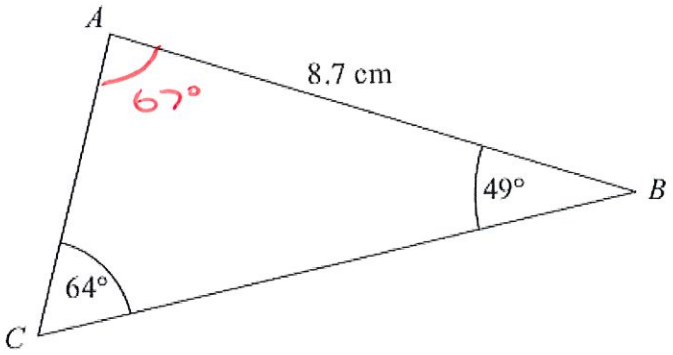
$BC^2 = A^2 + 5.8^2 - 2 \times A \times 5.8 \times \cos 100$

BC = 8.5190...

.....8.52..... cm

(Total for question = 5 marks)

2. Diagram **NOT** accurately drawn
ABC is a triangle.
AB = 8.7 cm.
Angle *ABC* = 49°.
Angle *ACB* = 64°.
Calculate the area of triangle *ABC*.
Give your answer correct to 3 significant figures.



$\frac{AC}{\sin 49} = \frac{8.7}{\sin 64}$

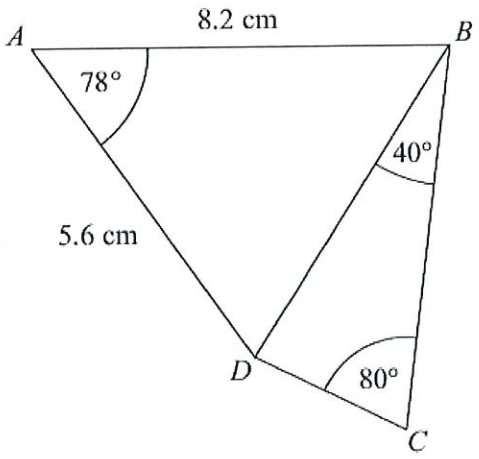
$\therefore$ AC = 7.305... (→A)

$0.5 \times A \times 8.7 \times \sin 67 = 29.251...$

.....29.6..... cm^2

(Total for Question is 5 marks)

3. *ABCD* is a quadrilateral.
Diagram **NOT** accurately drawn
Work out the length of *DC*.
Give your answer correct to 3 significant figures.



$BD^2 = 8.2^2 + 5.6^2 - 2 \times 8.2 \times 5.6 \times \cos 78$

$BD = 8.916...$ (→A)

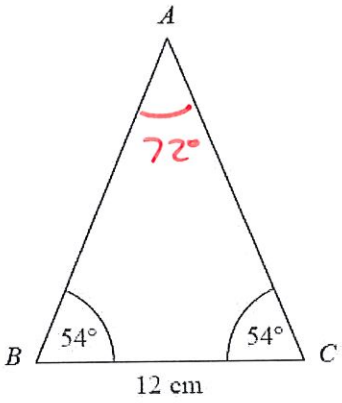
$\frac{DC}{\sin 40} = \frac{"A"}{\sin 80}$

$DC = 5.8198...$

5.82 ..... cm

(Total for Question is 6 marks)

4. Diagram **NOT** accurately drawn
ABC is an isosceles triangle.
Work out the area of the triangle.
Give your answer correct to 3 significant figures.



$\frac{AB}{\sin 54} = \frac{12}{\sin 72}$

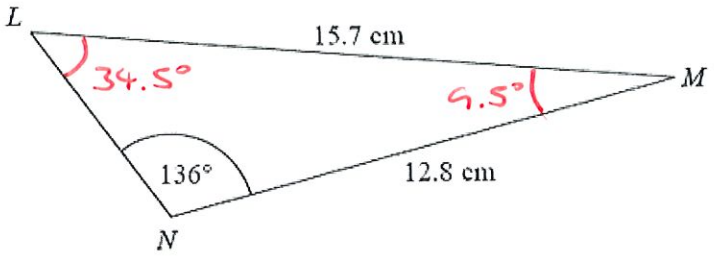
$AB = 10.207...$ (→A)

$0.5 \times "A"^2 \times \sin 72 = 49.549...$

49.5 ..... cm^2

(Total for Question is 4 marks)

5. Diagram **NOT** accurately drawn
The diagram shows triangle *LMN*.
Calculate the length of *LN*.
Give your answer correct to 3 significant figures.



$\frac{\sin L}{12.8} = \frac{\sin 136}{15.7}$

$\therefore L = 34.495...$ (→A)

$M = 9.504...$ (→B)

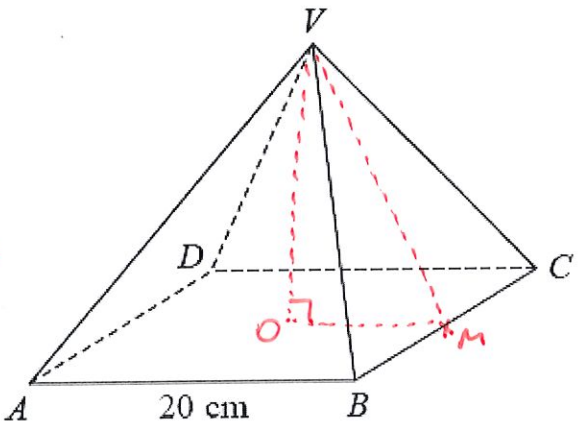
$LN^2 = 15.7^2 + 12.8^2 - 2 \times 15.7 \times 12.8 \times \cos "B"$

$LN = 3.731...$

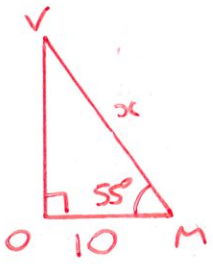
3.73 ..... cm

(Total for Question is 5 marks)

6. *VABCD* is a solid pyramid.
ABCD is a square of side 20 cm.
The angle between any sloping edge and the plane *ABCD* is 55°
Calculate the surface area of the pyramid.
Give your answer correct to 2 significant figures.



Base area : $20 \times 20 = 400 cm^2$



$x = 10 \div \cos 55$

$= 17.434...$ (→A)

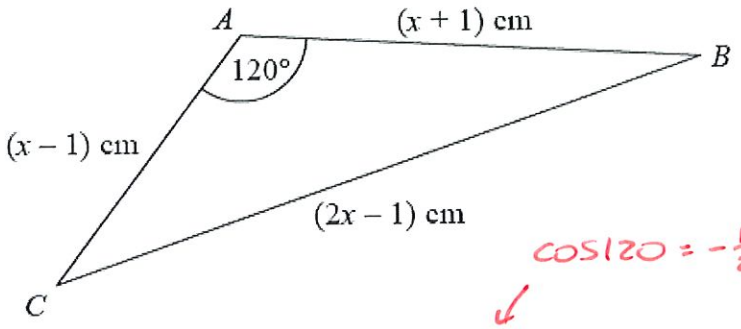
Area of triangle : $\frac{20 \times A}{2} = 174.34...$ (→B)

Total area : $400 + 4 \times B = 1097.37...$

1100 ..... cm^2

(Total for question = 5 marks)

7. The diagram shows triangle *ABC*.
The area of triangle *ABC* is $k\sqrt{3}$ cm^2.
Find the exact value of *k*.



$\cos 120 = -\frac{1}{2}$

$(2x-1)^2 = (x-1)^2 + (x+1)^2 - 2(x+1)(x-1)\cos 120$

$4x^2 - 4x + 1 = x^2 - 2x + 1 + x^2 + 2x + 1 + x^2 - 1$

$x^2 - 4x = 0$

$x = 0$ or 4

$\therefore x - 1 = 3$ and $x + 1 = 5$

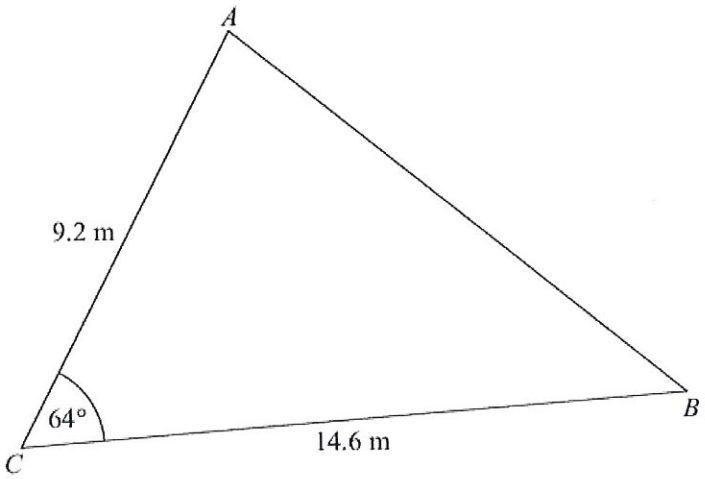
$\frac{1}{2} \times 3 \times 5 \times \sin 120 = \frac{15\sqrt{3}}{4}$

k = $\frac{15}{4}$

(Total for question = 7 marks)

8. Diagram **NOT** accurately drawn
AC = 9.2 m
BC = 14.6 m
Angle *ACB* = 64°

(a) Calculate the area of the triangle *ABC*.
Give your answer correct to 3 significant figures.



$0.5 \times 9.2 \times 14.6 \times \sin 64$

$= 60.363...$

60.4 m^2

(2)

(b) Calculate the length of *AB*.
Give your answer correct to 3 significant figures.

$AB^2 = 9.2^2 + 14.6^2 - 2 \times 9.2 \times 14.6 \times \cos 64$

$AB = 13.417...$

13.4

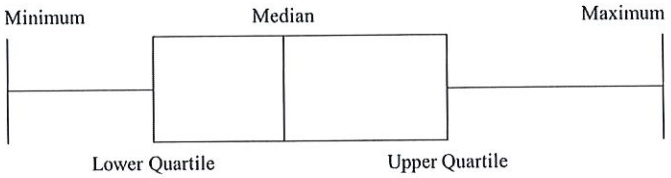
(3)

(Total for Question is 5 marks)

Cumulative frequency and box plots

Things to remember:

- Use a running total – adding on to complete the cumulative frequency column;
- Plot at the end of the group;
- Join up with a smooth curve;
- To find the median find the value half way down the cumulative frequency, draw across to the line and then vertically down to find the value – always show these working lines;
- To find the interquartile range find the upper quartile and the lower quartile and subtract them.
- To draw a box plot →
- When comparing box plots, use the median and the IQR and keep words consistent with the question.



Questions:

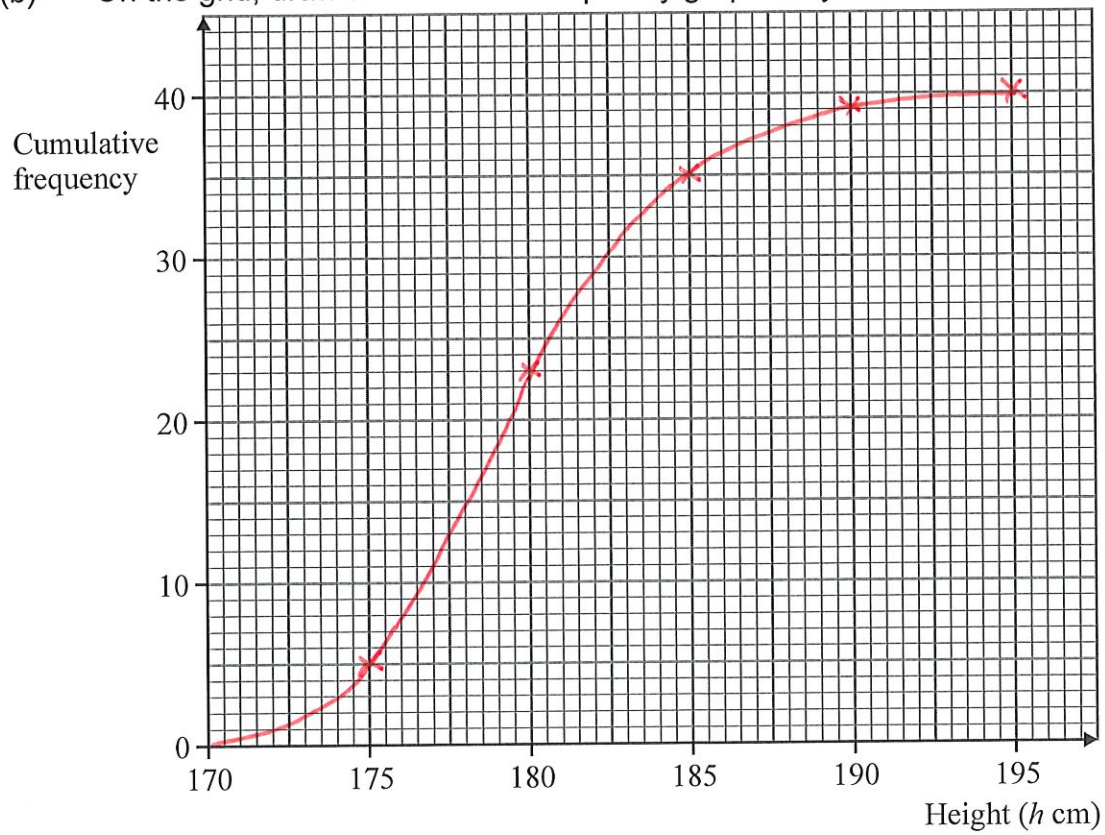
1. The table shows information about the heights of 40 bushes.

Height (h cm)	Frequency	Cumulative Frequency
$170 \leq h < 175$	5	5
$175 \leq h < 180$	18	23
$180 \leq h < 185$	12	35
$185 \leq h < 190$	4	39
$190 \leq h < 195$	1	40

(a) Complete the cumulative frequency table above.

(1)

(b) On the grid, draw a cumulative frequency graph for your table.



(2)

(Total 3 marks)

2. The table gives information about the ages of 160 employees of an IT company.

Age (A) in years	Frequency	Cumulative Frequency
$15 < A \leq 25$	44	44
$25 < A \leq 35$	56	100
$35 < A \leq 45$	34	134
$45 < A \leq 55$	19	153
$55 < A \leq 65$	7	160

(a) Write down the modal class interval.

$25 < A \leq 35$

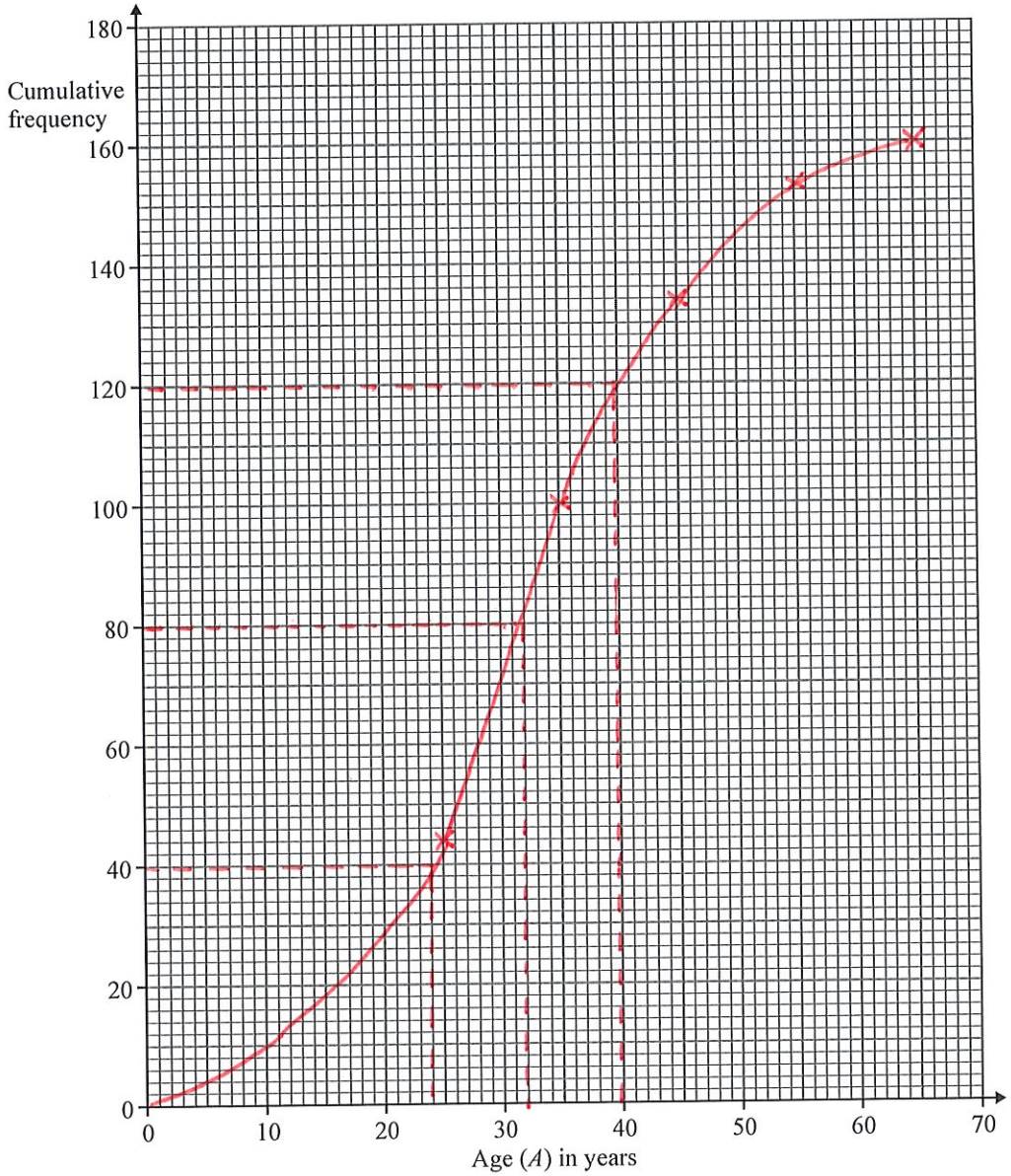
(1)

(b) Complete the cumulative frequency table.

(1)

(c) On the grid below, draw a cumulative frequency graph for your table.

(2)



(d) Use your graph to find an estimate for

(i) the median age of the employees,

32 years

(i) the interquartile range of the ages of the employees.

16 years

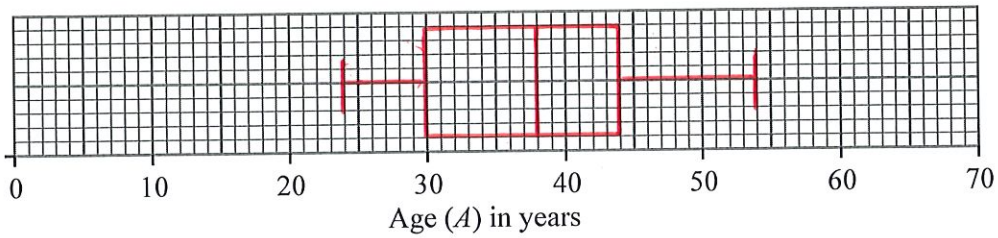
40 − 24

(3)

Another IT company has 80 employees. The age of the youngest employee is 24 years. The age of the oldest employee is 54 years. The median age is 38 years. The lower quartile age is 30 years. The upper quartile age is 44 years.

(e) On the grid below, draw a box plot to show information about the ages of the employees.

(2)



(Total 9 marks)

3. A company tested 100 batteries. The table shows information about the number of hours that the batteries lasted.

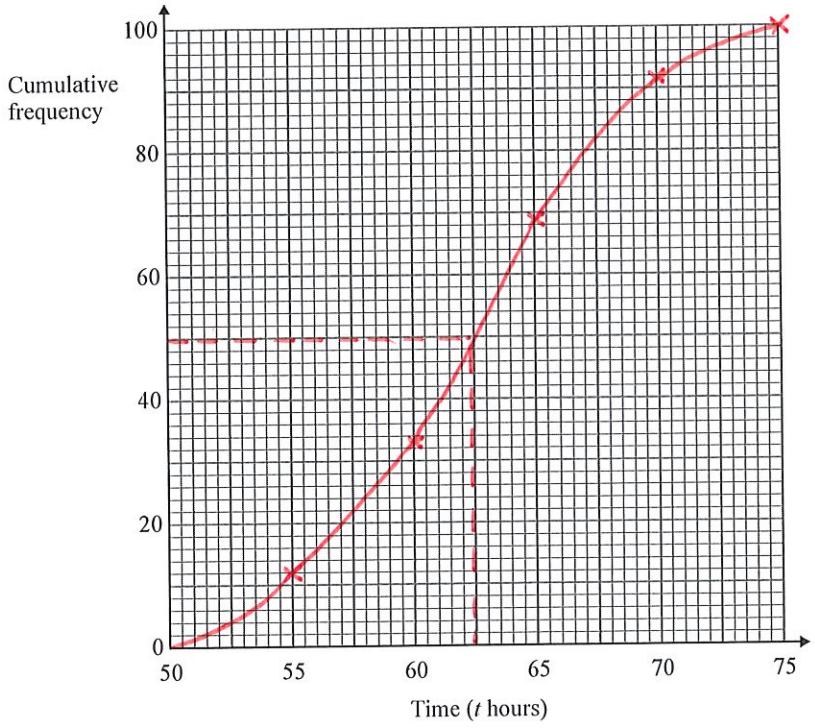
Time (t hours)	Frequency	Cumulative Frequency
$50 \leq t < 55$	12	12
$55 \leq t < 60$	21	33
$60 \leq t < 65$	36	69
$65 \leq t < 70$	23	92
$70 \leq t < 75$	8	100

(a) Complete the cumulative frequency table for this information.

(1)

(b) On the grid, draw a cumulative frequency graph for your completed table.

(2)



(c) Use your completed graph to find an estimate for the median time. You must state the units of your answer.

62.5

(2)

(Total 5 marks)

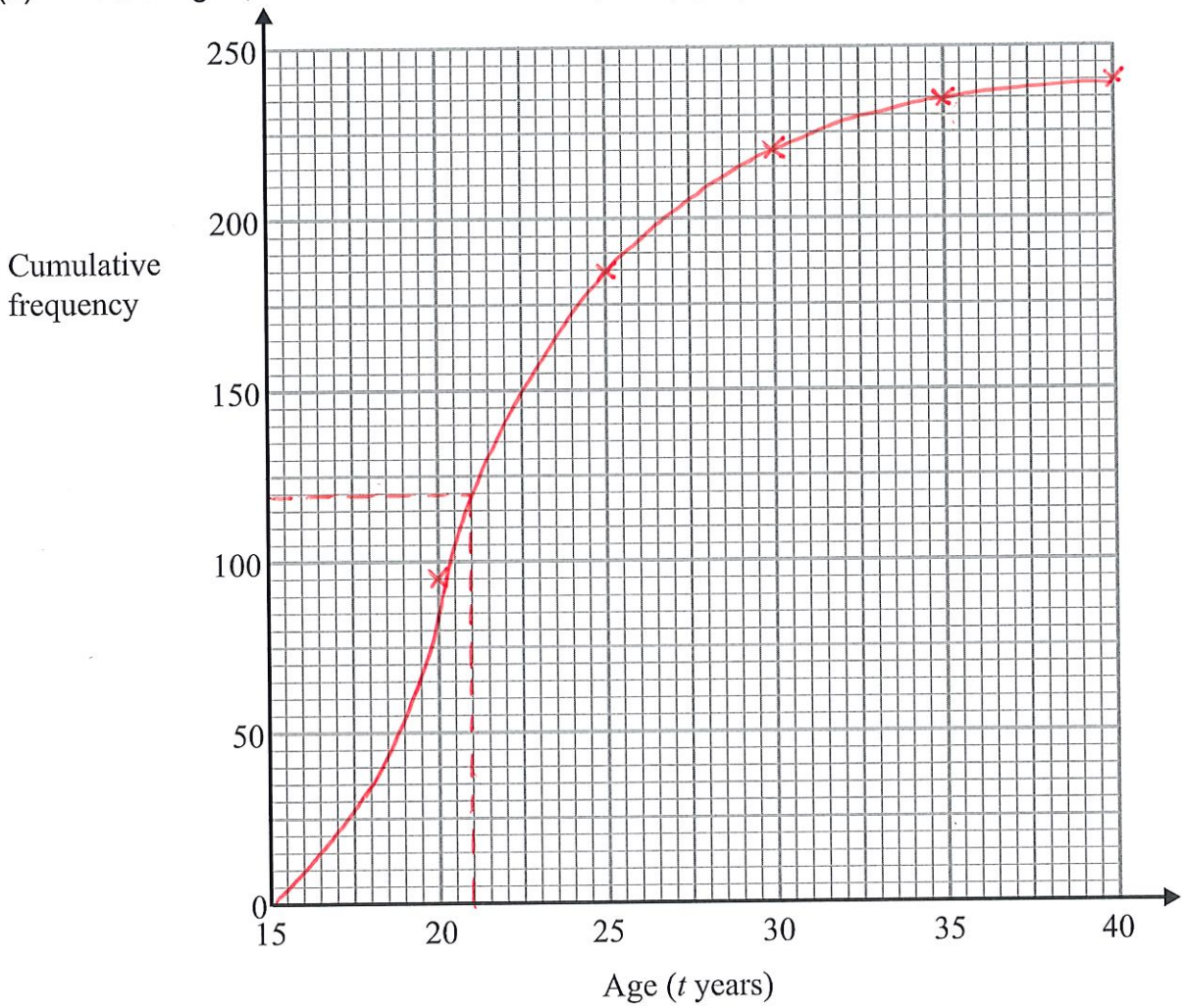
4. The table shows information about the ages of the 240 people at a club.

Age (t years)	Frequency	Cumulative Frequency
$15 \leq t < 20$	95	95
$20 \leq t < 25$	90	185
$25 \leq t < 30$	35	220
$30 \leq t < 35$	15	235
$35 \leq t < 40$	5	240

(a) Complete the cumulative frequency table.

(1)

(b) On the grid, draw the cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median age of the people.

21 years

(1)

(Total 4 marks)

5. An operator took 100 calls at a call centre. The table gives information about the time (t seconds) it took the operator to answer each call.

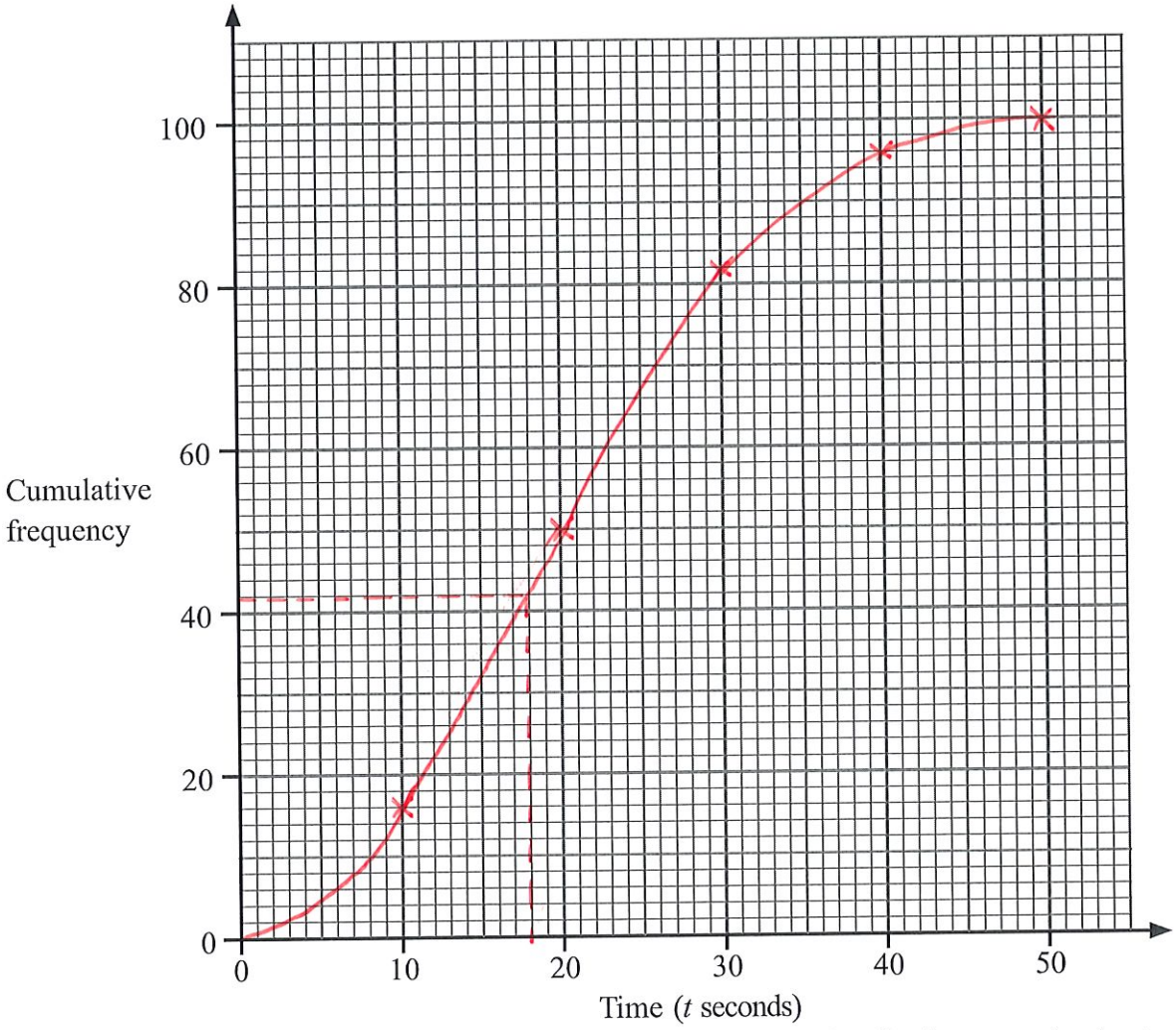
Time (t seconds)	Frequency	Cumulative Frequency
$0 < t \leq 10$	16	16
$10 < t \leq 20$	34	50
$20 < t \leq 30$	32	82
$30 < t \leq 40$	14	96
$40 < t \leq 50$	4	100

(a) Complete the cumulative frequency table.

(1)

(b) On the grid, draw a cumulative frequency graph for your table.

(2)



(c) Use your graph to find an estimate for the number of calls the operator took **more** than 18 seconds to answer.

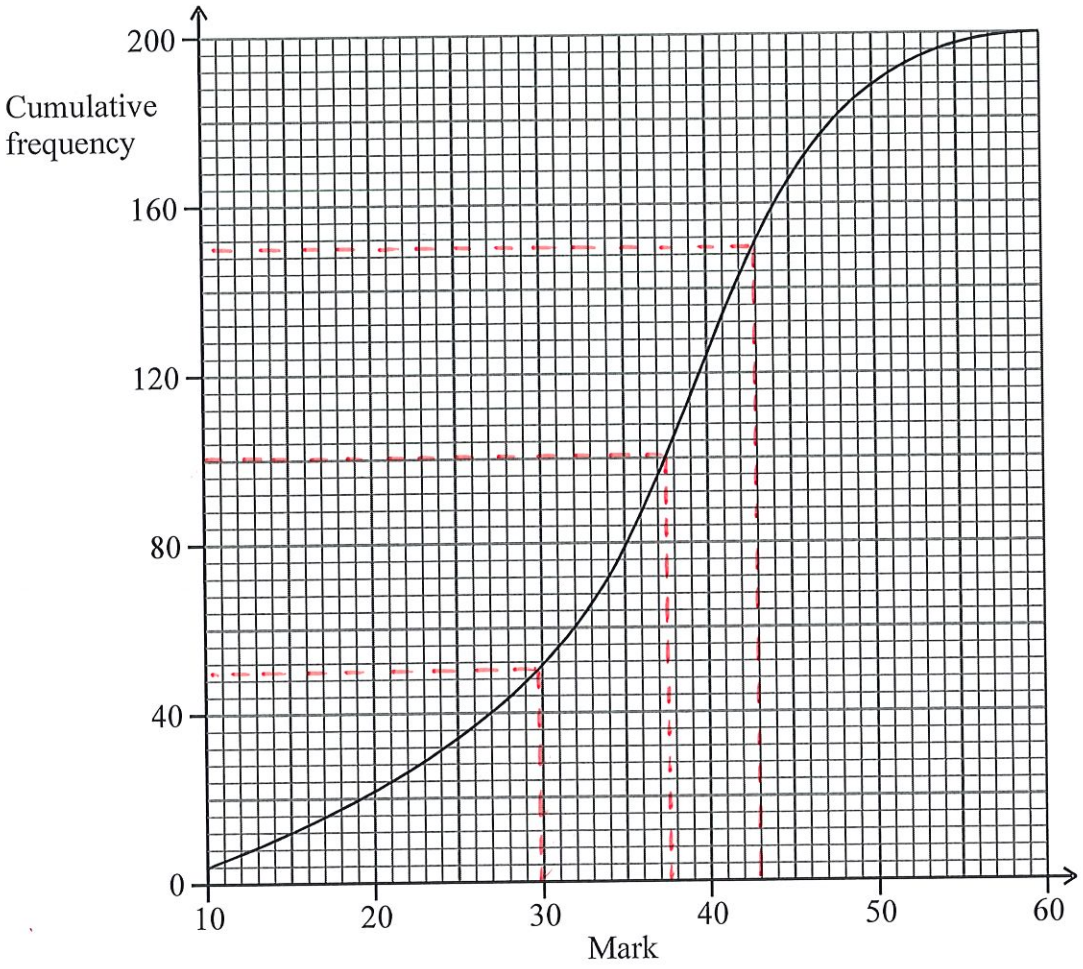
100 − 42

58

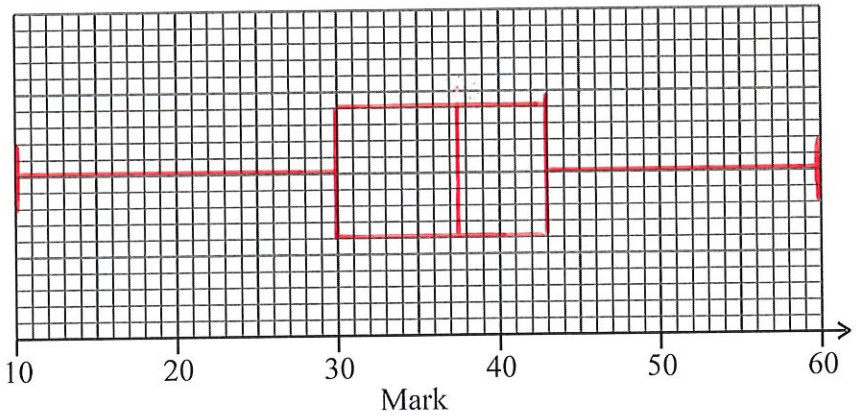
(2)

(Total 5 marks)

6. 200 students took a test. The cumulative frequency graph gives information about their marks.



The lowest mark scored in the test was 10.
The highest mark scored in the test was 60.
Use this information and the cumulative frequency graph to draw a box plot showing information about the students' marks.

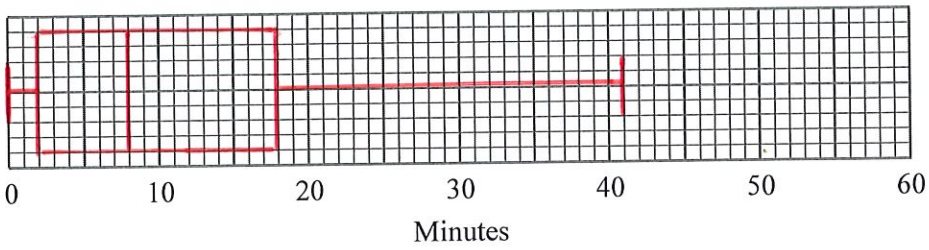


(Total 3 marks)

7. On Friday, Peter went to the airport.
He recorded the number of minutes that each plane was delayed.
He used his results to work out the information in this table.

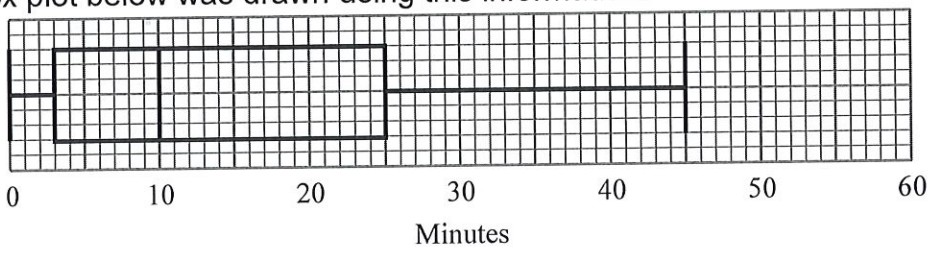
	Minutes
Shortest delay	0
Lower quartile	2
Median	8
Upper quartile	18
Longest delay	41

(a) On the grid, draw a box plot to show the information in the table.



(2)

Peter also went to the airport on Saturday.
He recorded the number of minutes that each plane was delayed.
The box plot below was drawn using this information.



(b) Make two comparisons between the distributions of plane delays on Friday and on Saturday.

On average, planes were delayed for longer on Saturday. Planes were more consistent in delays on Friday.

(2)
(Total 4 marks)

Histograms

Things to remember:

- Frequency = Frequency Density x Class Width;
- The y-axis will always be labelled "frequency density";
- The x-axis will have a continuous scale.

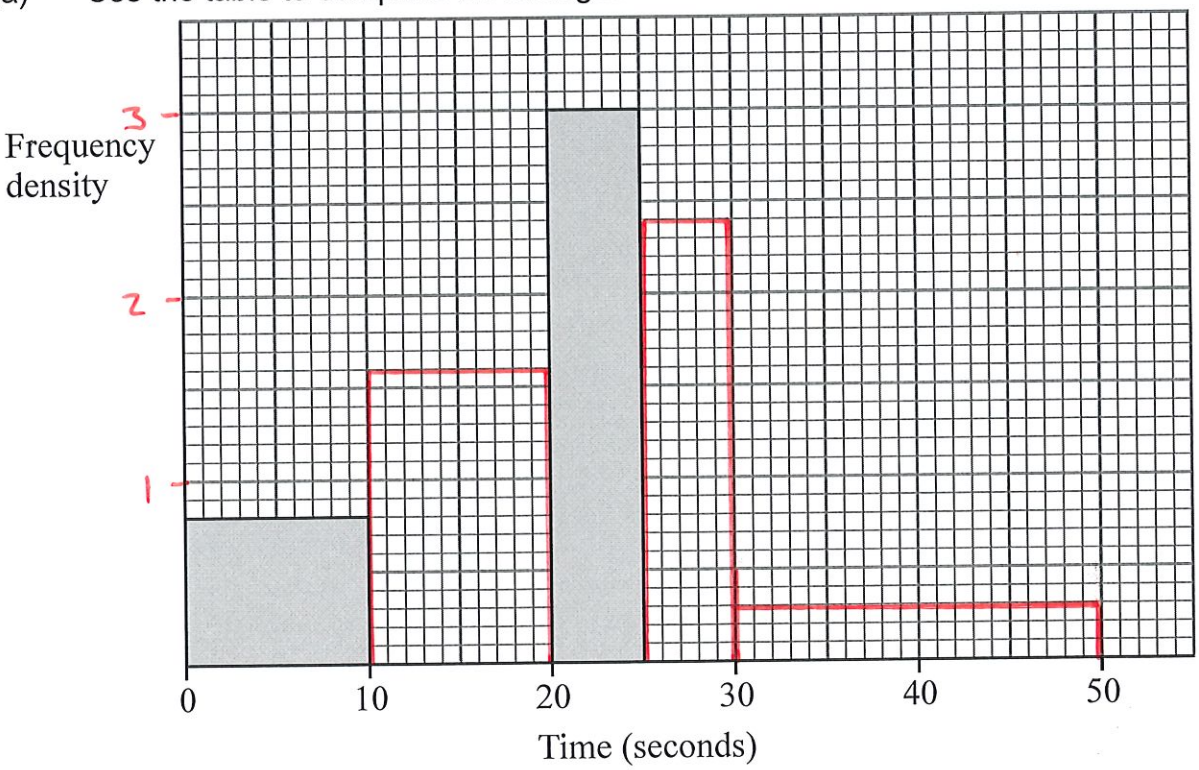
Questions:

1. One Monday, Victoria measured the time, in seconds, that individual birds spent on her bird table. She used this information to complete the frequency table.

Time (t seconds)	Frequency
$0 < t \leq 10$	8
$10 < t \leq 20$	16
$20 < t \leq 25$	15
$25 < t \leq 30$	12
$30 < t \leq 50$	6

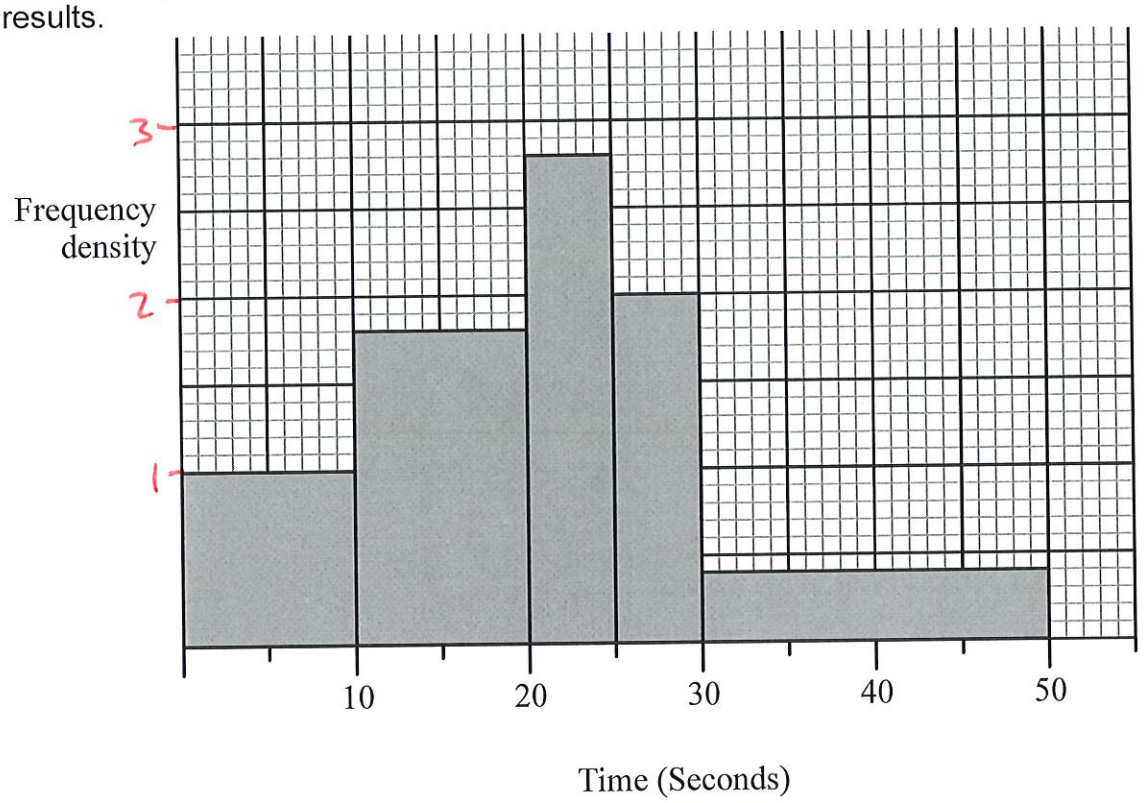
FD
8 ÷ 10 = 0.8
16 ÷ 10 = 1.6
15 ÷ 5 = 3
12 ÷ 5 = 2.4
6 ÷ 20 = 0.3

(a) Use the table to complete the histogram.



(3)

On Tuesday she conducted a similar survey and drew the following histogram from her results.

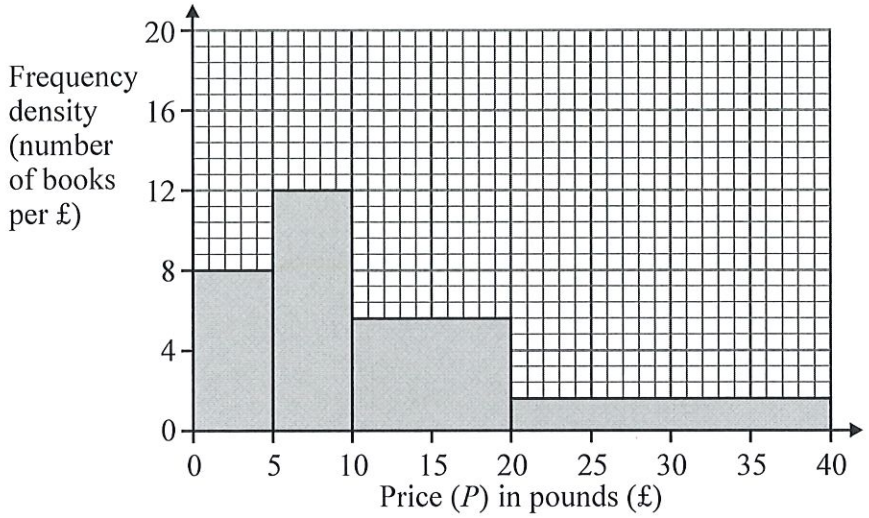


(b) Use the histogram for Tuesday to complete the table.

Time (t seconds)	Frequency	FD
$0 < t \leq 10$	10	$10 \div 10 = 1$
$10 < t \leq 20$	18	1.8
$20 < t \leq 25$	14	2.8
$25 < t \leq 30$	10	2
$30 < t \leq 50$	8	0.4

(2)
(Total 5 marks)

2. This histogram gives information about the books sold in a bookshop one Saturday.



(a) Use the histogram to complete the table.

Price (P) in pounds (£)	Frequency
$0 < P \leq 5$	40
$5 < P \leq 10$	60
$10 < P \leq 20$	56
$20 < P \leq 40$	32

FD
8
12
5.6
1.6

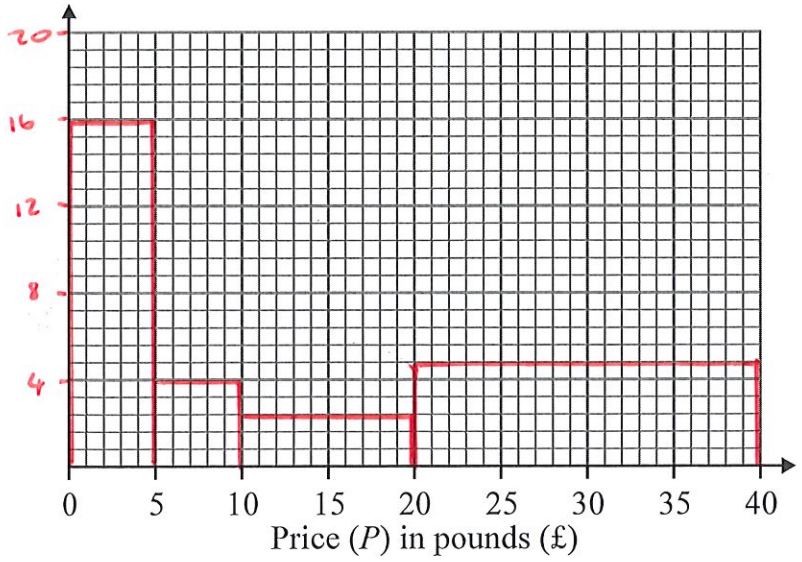
(2)

The frequency table below gives information about the books sold in a second bookshop on the same Saturday.

Price (P) in pounds (£)	Frequency
$0 < P \leq 5$	80
$5 < P \leq 10$	20
$10 < P \leq 20$	24
$20 < P \leq 40$	96

FD
$80 \div 5 = 16$
$20 \div 5 = 4$
$24 \div 10 = 2.4$
$96 \div 20 = 4.8$

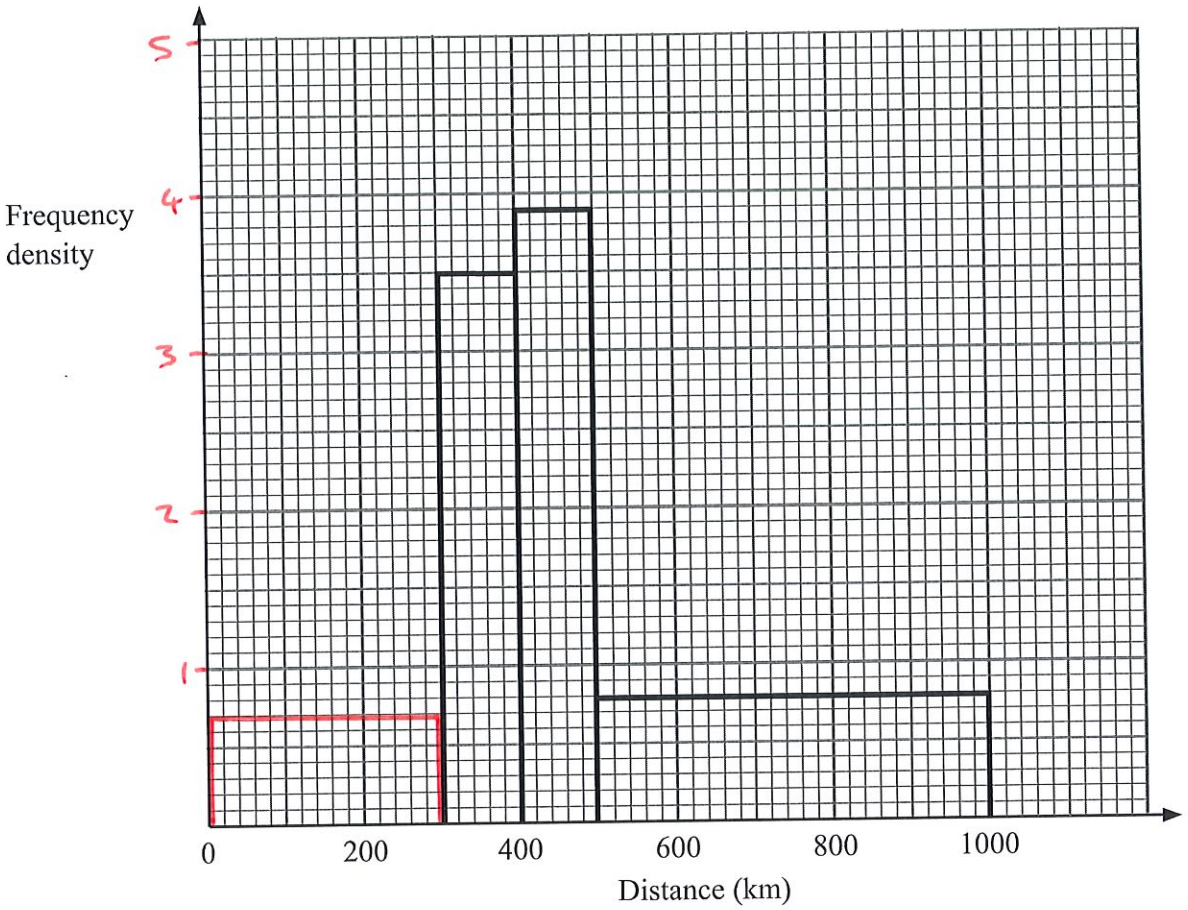
(b) On the grid below, draw a histogram to represent the information about the books sold in the second bookshop.



(3)

(Total 5 marks)

3. The incomplete table and histogram give some information about the distances walked by some students in a school in one year.



(a) Use the information in the histogram to complete the frequency table.

Distance (d) in km	Frequency
$0 < d \leq 300$	210
$300 < d \leq 400$	350
$400 < d \leq 500$	390
$500 < d \leq 1000$	400

FD
210 ÷ 300 = 0.7
350 ÷ 100 = 3.5
3.9
0.8

(2)

(b) Use the information in the table to complete the histogram.

(1)

(Total 3 marks)

4. The incomplete histogram and table show information about the weights of some containers.

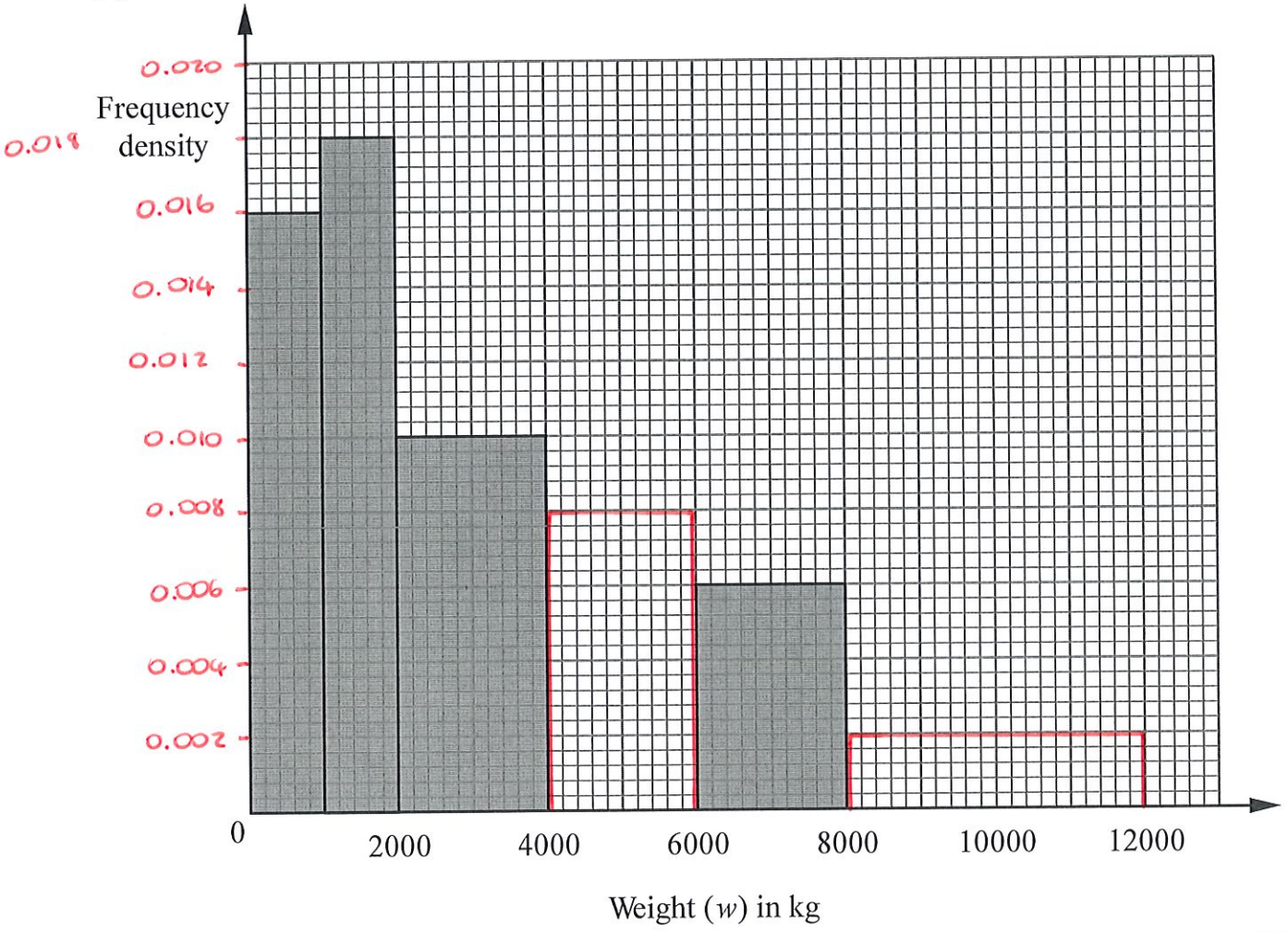
Weight (w) in kg	Frequency
$0 < w \leq 1000$	16
$1000 < w \leq 2000$	18
$2000 < w \leq 4000$	20
$4000 < w \leq 6000$	16
$6000 < w \leq 8000$	12
$8000 < w \leq 12000$	8

16 ÷ 1000 = 0.016
0.018
0.01
0.008
0.006
0.002

(a) Use the information in the histogram to complete the table.

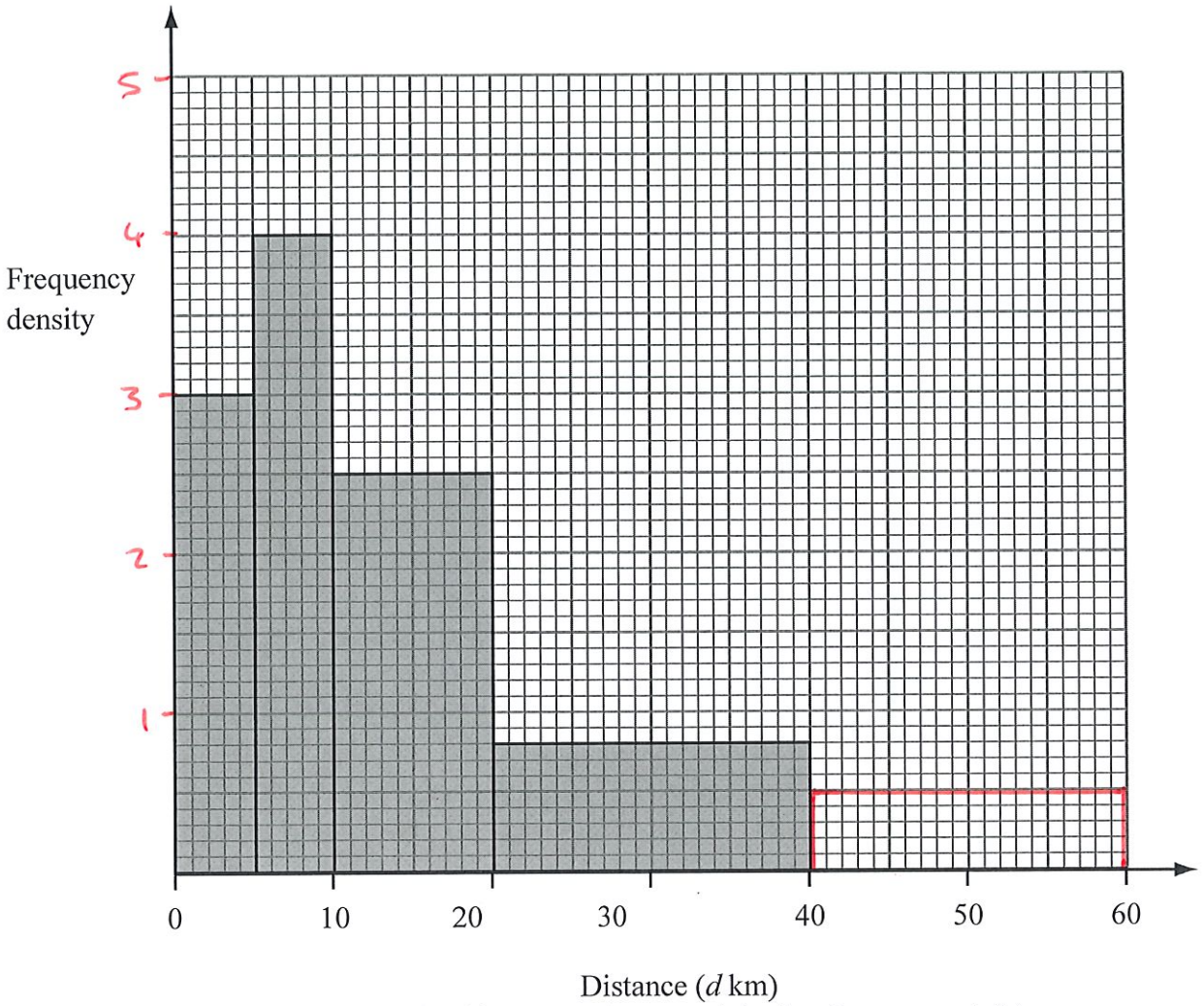
(2)

(b) Use the information in the table to complete the histogram.



(2)
(Total 4 marks)

5. The incomplete histogram and table give some information about the distances some teachers travel to school.



(a) Use the information in the histogram to complete the frequency table.

Distance ($d$km)	Frequency	FD
$0 < d \leq 5$	15	$15 \div 5 = 3$
$5 < d \leq 10$	20	4
$10 < d \leq 20$	25	2.5
$20 < d \leq 40$	16	0.8
$40 < d \leq 60$	10	0.5

(2)

(b) Use the information in the table to complete the histogram.

(1)

(Total 3 marks)

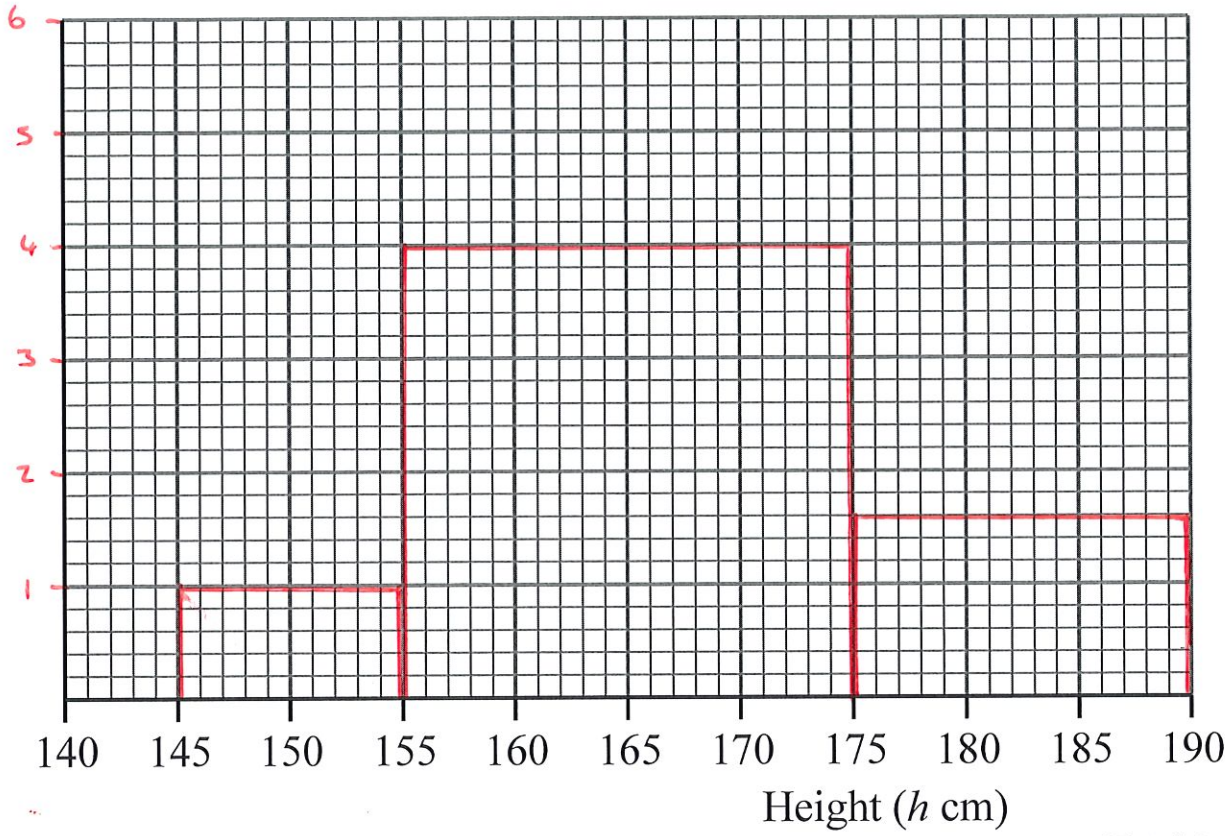
6. The table gives information about the heights, in centimetres, of some 15 year old students.

Height (h cm)	$145 < h \leq 155$	$155 < h \leq 175$	$175 < h \leq 190$
Frequency	10	80	24

FD 1 4 1.6

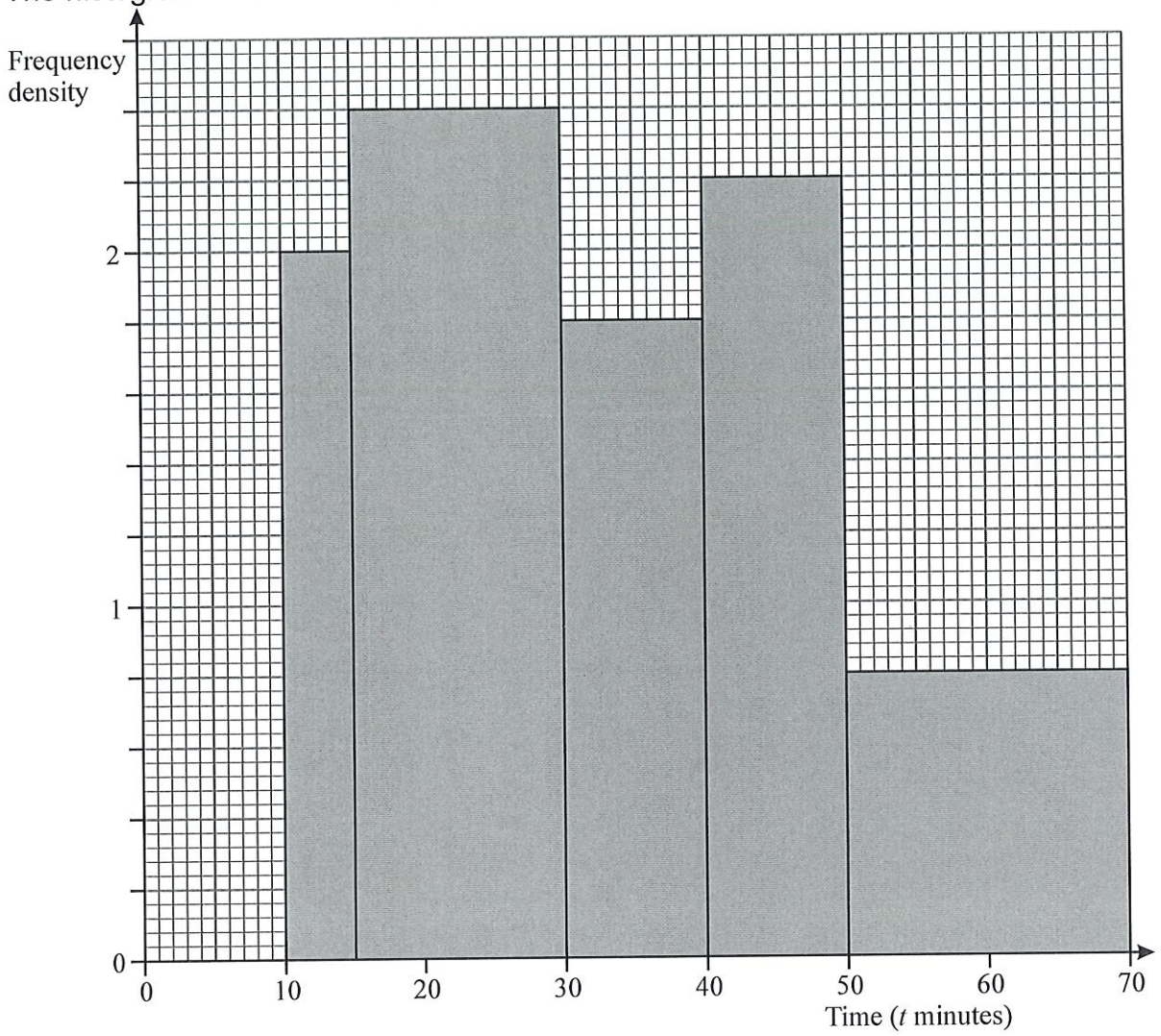
Use the table to draw a histogram.

FD



(Total 3 marks)

7. A teacher asked some year 10 students how long they spent doing homework each night. The histogram was drawn from this information.



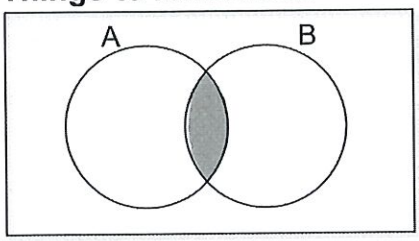
Use the histogram to complete the table.

Time (t minutes)	Frequency	FD
$10 \leq t < 15$	10	2
$15 \leq t < 30$	36	2.4
$30 \leq t < 40$	18	1.8
$40 \leq t < 50$	22	2.2
$50 \leq t < 70$	16	0.8

(Total 2 marks)

Set Theory

Things to remember:

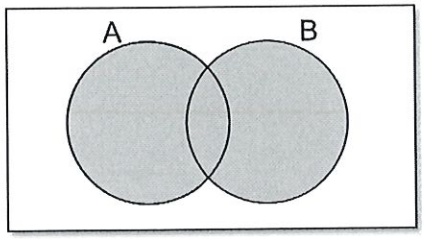


The **intersection** is where two sets overlap.

$A \cap B$

This means **A and B**.

AND rule

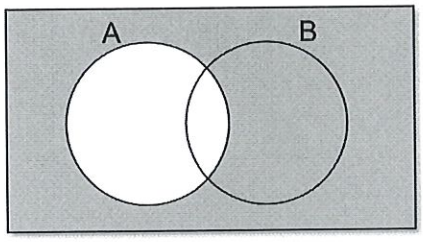


If you put two sets together, you get the **union**.

$A \cup B$

This means **A or B**.

OR rule



The **complement of A** is the region that is not A.

A'

This means **not A**.

Questions:

1.

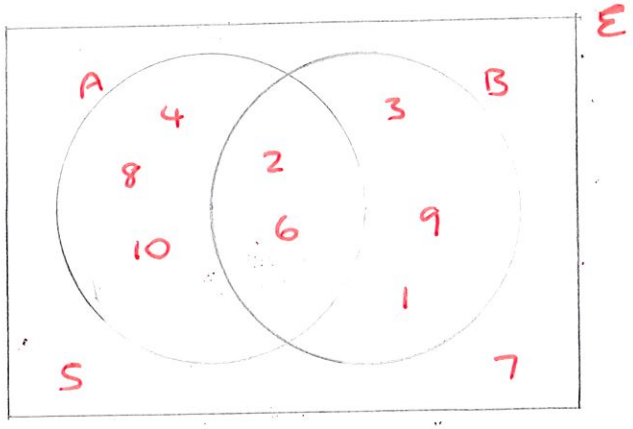
$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{\text{multiples of } 2\}$

$A \cap B = \{2, 6\}$

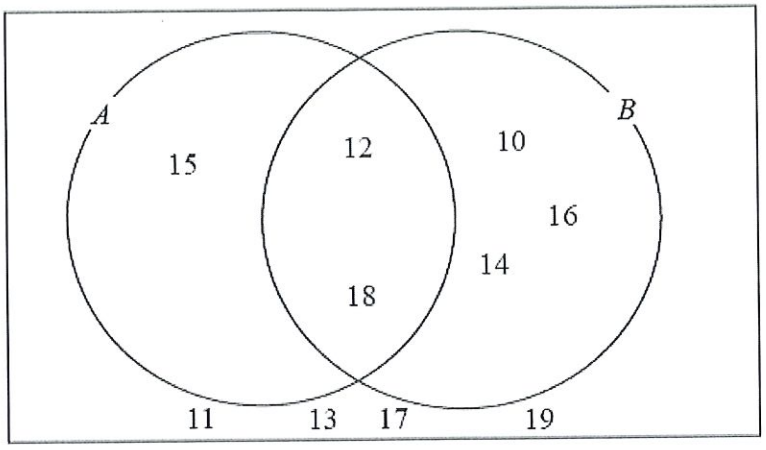
$A \cup B = \{1, 2, 3, 4, 6, 8, 9, 10\}$

Draw a Venn diagram for this information.



(Total for question is 4 marks)

2. Here is a Venn diagram.



(a) Write down the numbers that are in set

(i) $A \cup B$

{10, 12, 14, 15, 16, 18}

(ii) $A \cap B$

{12, 18}

(2)

One of the numbers in the diagram is chosen at random.

(b) Find the probability that the number is in set A'

$1 - \frac{3}{10}$

$\frac{7}{10}$

(2)

(Total for question = 4 marks)

T C M

4. Sami asked 50 people which drinks they liked from tea, coffee and milk.

3. All 50 people like at least one of the drinks

19 people like all three drinks.

16 people like tea and coffee but do not like milk.

21 people like coffee and milk.

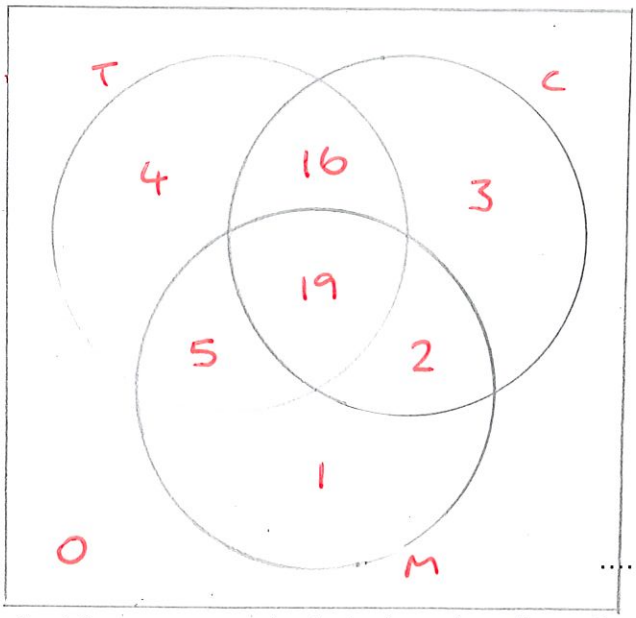
24 people like tea and milk.

40 people like coffee.

1 person likes only milk.

Sami selects at random one of the 50 people.

(a) Work out the probability that this person likes tea.



P(Like tea) = $\frac{44}{50}$

(4)

(b) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.

$\frac{21}{44}$

(2)

(Total for question = 6 marks)

Proportion

Things to remember:

- Start by checking the question for squares, cubes and roots;
- "x is directly proportional to y" looks like **x α y** or **x = ky**
- "x is inversely proportional to y" looks like **x α** $\frac{1}{y}$ or **x =** $\frac{k}{y}$

Questions:

1. The shutter speed, *S*, of a camera varies inversely as the square of the aperture setting, *f*.
When $f = 8$, $S = 125$

(a) Find a formula for *S* in terms of *f*.

$S \propto \frac{1}{f^2}$ $\quad k = 8000$

$S = \frac{k}{f^2}$

$125 = \frac{k}{8^2}$

$S = \frac{8000}{f^2}$

(3)

(b) Hence, or otherwise, calculate the value of *S* when $f = 4$

$S = \frac{8000}{4^2}$

S = 500

(1)

(Total 4 marks)

2. In a factory, chemical reactions are carried out in spherical containers.
The time, *T* minutes, the chemical reaction takes is directly proportional to the square of the radius, *R* cm, of the spherical container.
When $R = 120$, $T = 32$
Find the value of *T* when $R = 150$

$T \propto R^2$

$T = kR^2$

$32 = 120^2 k$

$k = \frac{1}{450}$

$T = \frac{R^2}{450}$ $\quad T = \frac{150^2}{450}$

T = 50

(Total 4 marks)

3. d is directly proportional to the square of t.
$d = 80$ when $t = 4$

(a) Express d in terms of t.

$d \propto t^2$ $\quad k = 5$

$d = kt^2$

$80 = 4^2 k$

$d = 5t^2$

(3)

(b) Work out the value of d when $t = 7$

$d = 5 \times 7^2$

$d =$ 245

(1)

(c) Work out the positive value of t when $d = 45$

$45 = 5t^2$

$9 = t^2$

$t =$ 3

(2)

(Total 6 marks)

4. The distance, D, travelled by a particle is directly proportional to the square of the time, t, taken. When $t = 40$, $D = 30$

(a) Find a formula for D in terms of t.

$D \propto t^2$

$D = kt^2$

$30 = 40^2 k$

$k = \frac{3}{160}$

$D =$ $\frac{3t^2}{160}$

(3)

(b) Calculate the value of D when $t = 64$

$D = \frac{3 \times 64^2}{160}$

76.8

(1)

(c) Calculate the value of t when $D = 12$
Give your answer correct to 3 significant figures.

$12 = \frac{3t^2}{160}$

$640 = t^2$

$t = 25.298...$

25.3

(2)

(Total 6 marks)

5. The time, T seconds, it takes a water heater to boil some water is directly proportional to the mass of water, m kg, in the water heater. When $m = 250$, $T = 600$

(a) Find T when $m = 400$

$T \propto m$

$T = km$

$600 = 250k$

$k = 2.4$

$T = 2.4m$

$T = 2.4 \times 400$

$T =$ 960

(3)

The time, T seconds, it takes a water heater to boil a constant mass of water is inversely proportional to the power, P watts, of the water heater.

When $P = 1400$, $T = 360$

(b) Find the value of T when $P = 900$

$T \propto \frac{1}{P}$

$360 = \frac{k}{1400}$

$k = 504000$

$T = \frac{504000}{P}$

$T = \frac{504000}{900}$

$T =$ 560

(3)

(Total 6 marks)

6. A ball falls vertically after being dropped.

The ball falls a distance d metres in a time of t seconds.

d is directly proportional to the square of t.

The ball falls 20 metres in a time of 2 seconds.

(a) Find a formula for d in terms of t.

$d \propto t^2$

$d = kt^2$

$20 = 2^2k$

$k = 5$

$d =$ $5t^2$

(3)

(b) Calculate the distance the ball falls in 3 seconds.

$d = 5 \times 3^2$

45 m

(1)

(c) Calculate the time the ball takes to fall 605 m.

$605 = 5t^2$

$121 = t^2$

$t = \pm 11$

11 seconds

(3)

(Total 7 marks)

7. In a spring, the tension (T newtons) is directly proportional to its extension (x cm). When the tension is 150 newtons, the extension is 6 cm.

(a) Find a formula for T in terms of x.

$T \propto x$

$T = kx$

$150 = 6k$

$k = 25$

$T =$ $25x$

(3)

(b) Calculate the tension, in newtons, when the extension is 15 cm.

$T = 25 \times 15$

375 newtons

(1)

(c) Calculate the extension, in cm, when the tension is 600 newtons.

$600 = 25x$

24 cm

(1)

(Total 5 marks)

8. f is inversely proportional to d.

When $d = 50$, $f = 256$

Find the value of f when $d = 80$

$f \propto \frac{1}{d}$

$f = \frac{k}{d}$

$256 = \frac{k}{50}$

$k = 12800$

$f = \frac{12800}{d}$

$f = \frac{12800}{80}$

$f =$ 160

(Total 3 marks)

Percentages – compound interest

Things to remember:

- New amount = original amount x multiplier^n ← Number of years

Questions:

1. Henry invests £4500 at a compound interest rate of 5% per annum.
At the end of *n* complete years the investment has grown to £5469.78.
Find the value of *n*.

4500×1.05^n

$n = 4$

(Total 2 marks)

2. Bill buys a new machine.
The value of the machine depreciates by 20% each year.

(a) Bill says 'after 5 years the machine will have no value'.
Bill is **wrong.** Explain why.

Bill has used the same 20% of the original amount. The 20% will decrease in value as the machine does.

(1)

Bill wants to work out the value of the machine after 2 years.

(b) By what single decimal number should Bill multiply the value of the machine when new?

0.8^2

0.64

(2)

(Total 3 marks)

3. Gwen bought a new car. Each year, the value of her car depreciated by 9%.
Calculate the number of years after which the value of her car was 47% of its value when new.

$0.9^n = 0.47$

7 years

(Total 3 marks)

4. The value of a car depreciates by 35% each year.
At the end of 2007 the value of the car was £5460
Work out the value of the car at the end of 2006

$5460 \div 0.65$

£ 8400

(Total 3 marks)

5. Toby invested £4500 for 2 years in a savings account.
He was paid 4% per annum compound interest.

(a) How much did Toby have in his savings account after 2 years?

4500×1.04^2

£ 4867.20

(3)

Jaspir invested £2400 for *n* years in a savings account.
He was paid 7.5% per annum compound interest.
At the end of the *n* years he had £3445.51 in the savings account.

(b) Work out the value of *n*.

$2400 \times 1.075^n = 3445.51$

$n = 5$

(2)

(Total 5 marks)

6. Mario invests £2000 for 3 years at 5% per annum **compound** interest.
Calculate the value of the investment at the end of 3 years.

2000×1.05^3

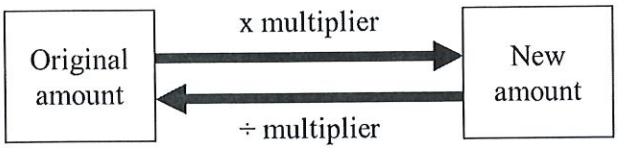
£ 2315.25

(Total 3 marks)

Percentages – reverse

Things to remember:

- Work out what the multiplier would have been;



Questions:

1. Loft insulation reduces annual heating costs by 20%.
After he insulated his loft, Curtley's annual heating cost was £520.
Work out Curtley's annual heating cost would have been, if he had not insulated his loft.

520 ÷ 0.8

£ 650
(Total 3 marks)

2. In a sale, normal prices are reduced by 20%.

SALE - 20% OFF

Andrew bought a saddle for his horse in the sale.
The sale price of the saddle was £220.
Calculate the normal price of the saddle.

220 ÷ 0.8

£ 275
(Total 3 marks)

3. Hajra's weekly pay this year is £240
This is 20% more than her weekly pay last year.
Bill says 'This means Hajra's weekly pay last year was £192'.
Bill is wrong,

(a) Explain why.

Bill has taken the new amount to be 100%, not the original amount.
(1)

(b) Work out Hajra's weekly pay last year.

240 ÷ 1.2

£ 200
(2)

(Total 3 marks)

4. The price of all rail season tickets to London increased by 4%.

(a) The price of a rail season ticket from Cambridge to London increased by £121.60
Work out the price before this increase.

4% = 121.6

100% = 3040

£ 3040

(2)

(b) After the increase, the price of a rail season ticket from Brighton to London was £2828.80
Work out the price before this increase.

2828.80 ÷ 1.04

£ 2720

(3)

(Total 5 marks)

5. In a sale, normal prices are reduced by 25%.
The sale price of a saw is £12.75
Calculate the normal price of the saw.

12.75 ÷ 0.75

£ 17

(Total 3 marks)

6. In a sale, normal prices are reduced by 12%.
The sale price of a DVD player is £242.
Work out the normal price of the DVD player.

242 ÷ 0.88

£ 275

(Total 3 marks)

7. A garage sells cars.
It offers a discount of 20% off the normal price for cash.
Dave pays £5200 cash for a car.
Calculate the normal price of the car.

5200 ÷ 0.8

£ 6500

(Total 3 marks)

Useful websites:

www.mathswatchvle.com

www.methodmaths.com

www.hegartymaths.com

www.mymaths.co.uk

www.drfrost.com

www.bbc.co.uk/schools/gcsebitesize/maths

Remember: Do your best;
it is all you can do ☺